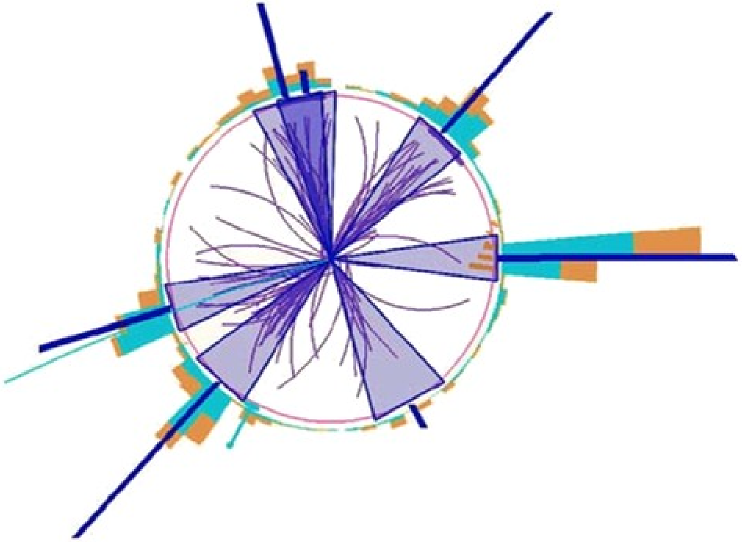
Giovanni Stagnitto

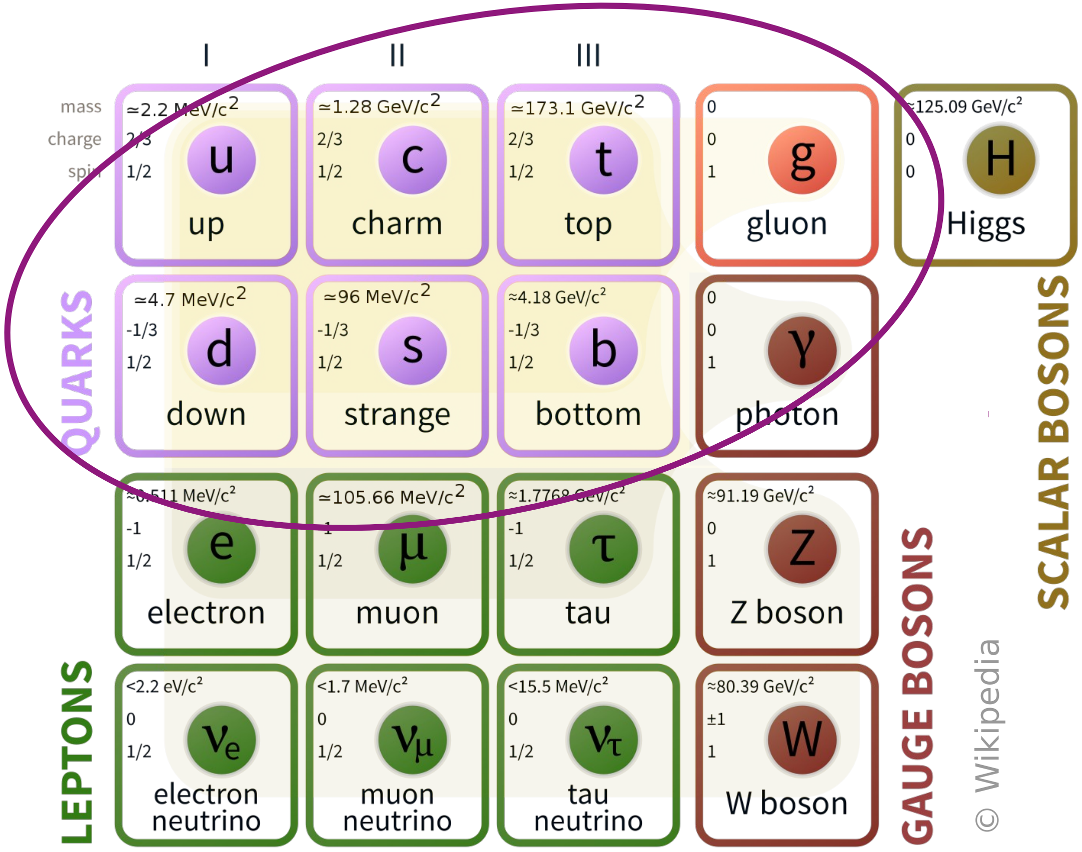
PhD Student in co-tutorship between
Università degli Studi di Milano & *INFN* (Milano) – Advisor: S. Forte
Université Paris Diderot & *LPTHE* (Paris) – Advisors: M. Cacciari, G. Soyez

Issues with the single jet inclusive cross section



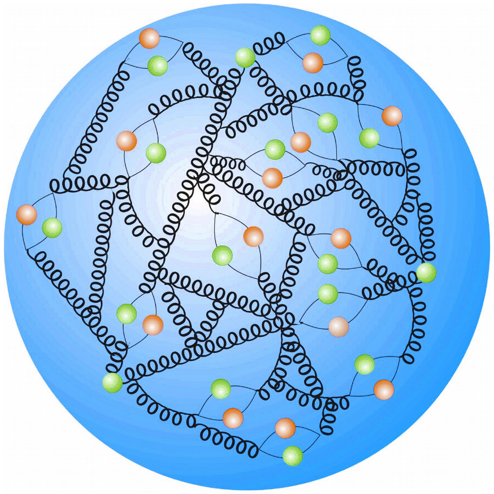
Workshop First-Year students – 9th & 10th October 2018 – Milano

Preamble: the strong world of QCD



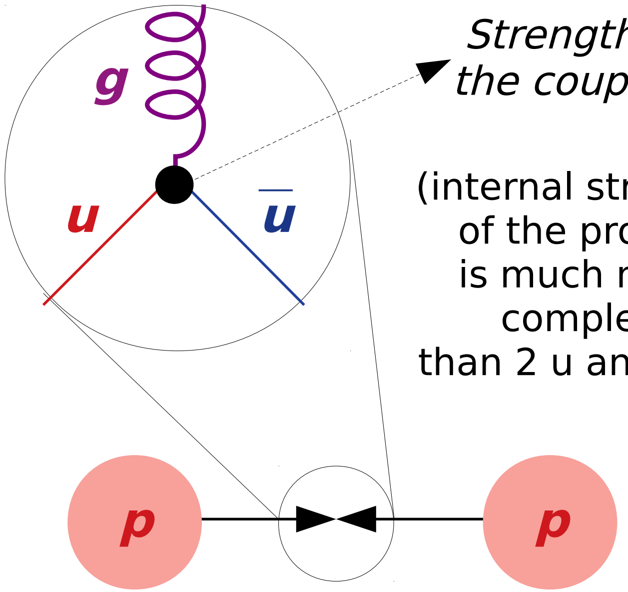
QCD = quantum chromodynamics

Quarks + **gluon** (the mediator of the strong force, from the Latin *gluten*) carry the color degree of symmetry (R, G, B)



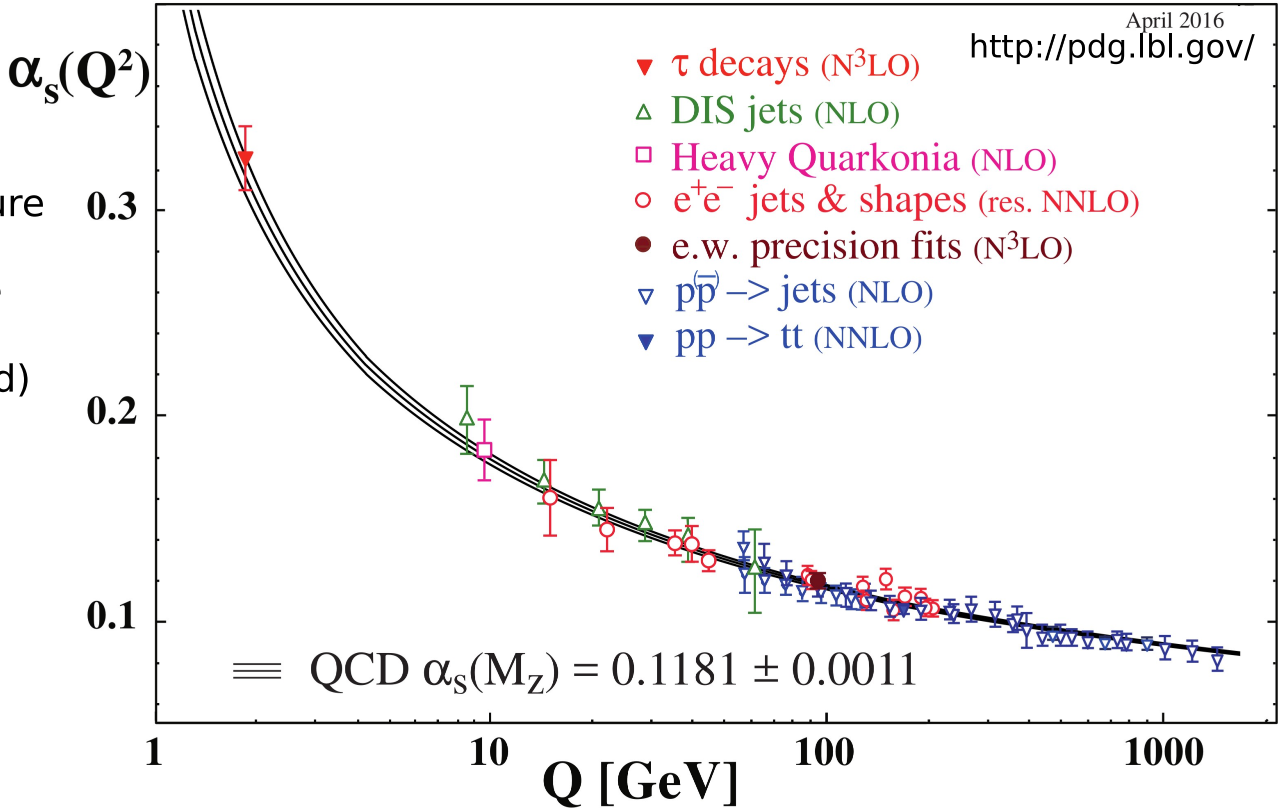
At ordinary energies QCD very strong → confinement into **hadrons** (color-neutral)

Preamble: the less strong world of perturbative QCD



Strength of the coupling

(internal structure of the proton is much more complex than 2 u and 1 d)

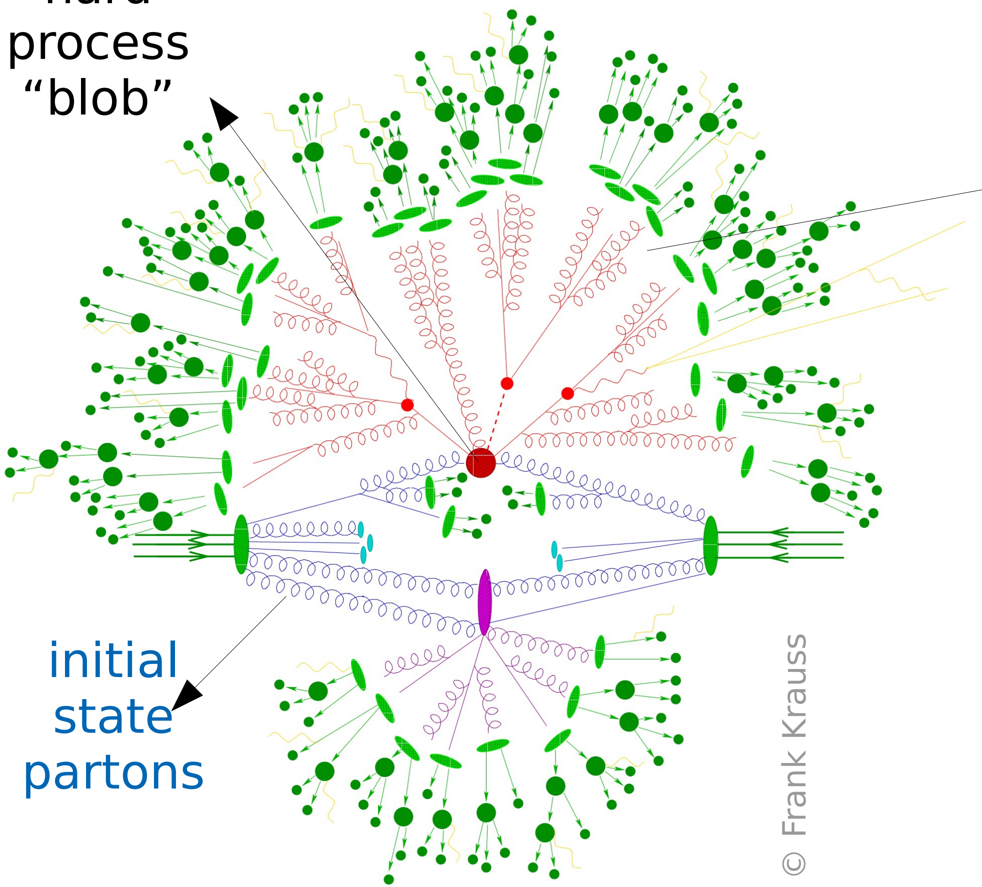


energy scale of the process

Large Hadron Collider (LHC)
@ CERN
is a proton-proton collider
at 13 TeV center-of-mass energy

The big picture of an event

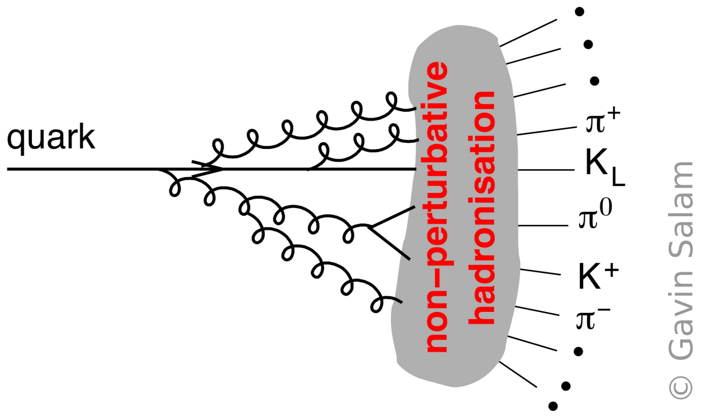
hard
process
"blob"



initial
state
partons

Scattering of two protons = big mess!

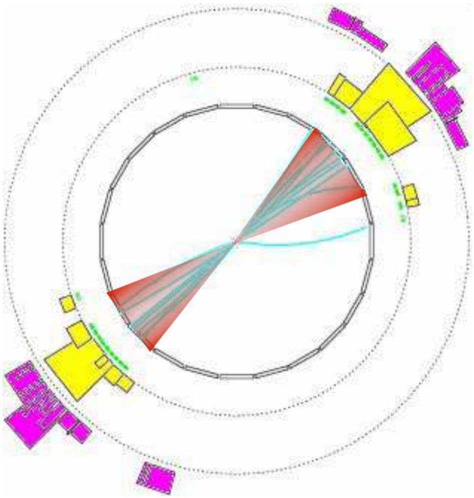
final state partons (colored!) →
→ "hadronization" → hadrons



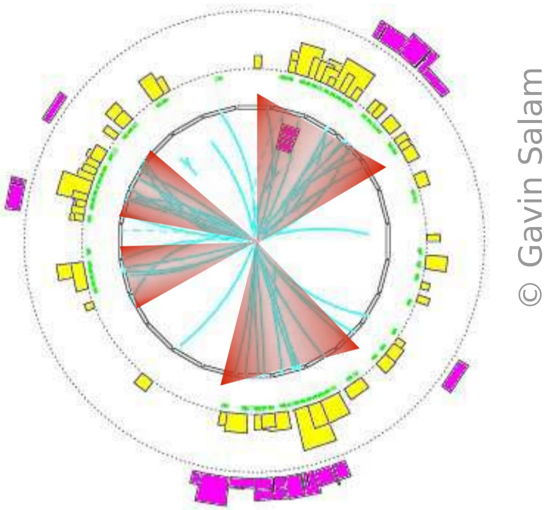
Jet = "collimated" bunch of hadrons

Why jets and not partons?

When talking about QCD in the final state, we can see only jets (confinement!)
Jets as a proxy for partons → study of QCD "blob"
BUT a proper definition is needed

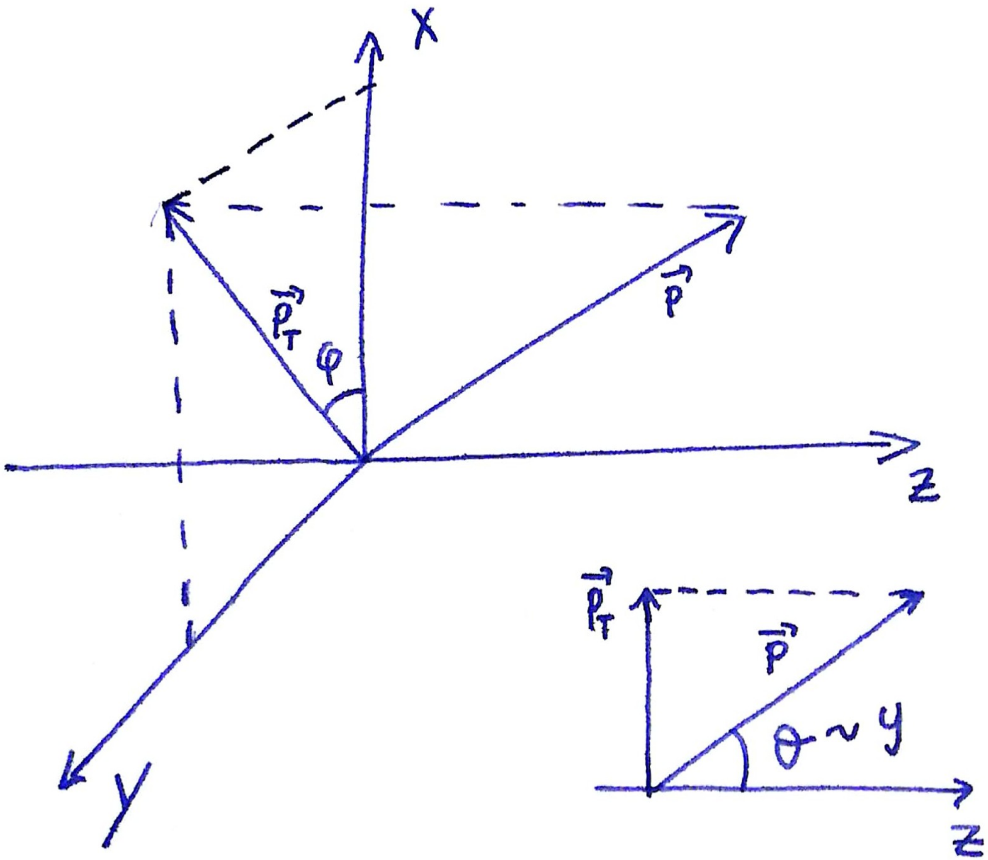


2 clear jets



3 jets?
or 4 jets?

An interlude on collider kinematics



$$(p_x, p_y, p_z) \to (p_t, y, \varphi)$$

p_t *transverse momentum*: projection of momentum on plane transverse to beam axis

y *rapidity*: related to angle formed with the beam axis

$$y = 0 \leftrightarrow \theta = 90$$

$$y = \pm\infty \leftrightarrow \theta = 0, 180$$

φ *azimuth angle*: rotation around beam axis

An example: anti-kt jet clustering algorithm

Distance between particle i and j:

$$d_{ij} = \frac{1}{\max(p_{ti}^2, p_{tj}^2)} \frac{R_{ij}^2}{R^2}, \quad d_{iB} = \frac{1}{p_{ti}^2}$$

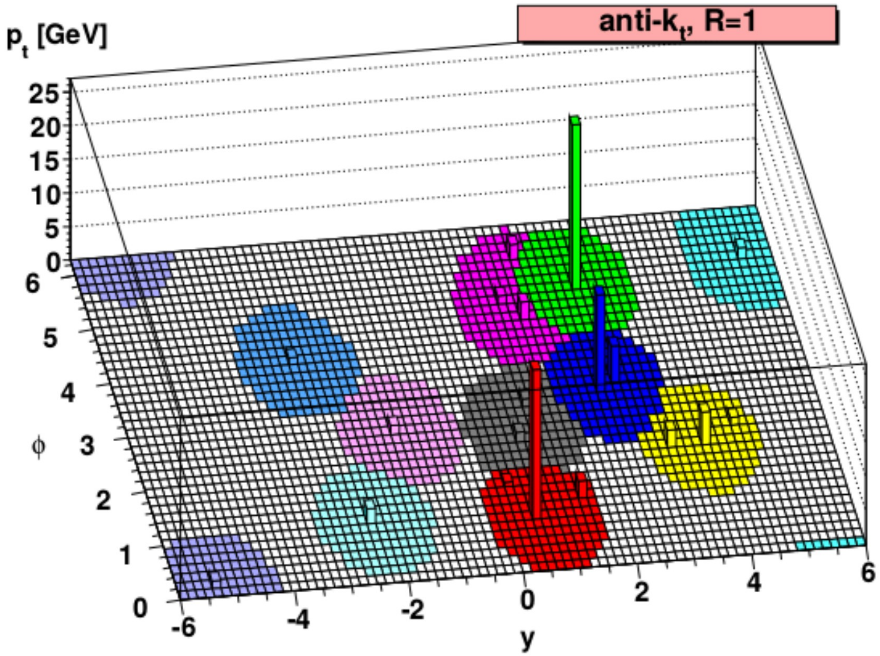
where the angular distance is defined as:

$$R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$$

R is the *radius* of the jet

If the smallest is a d_{ij}, recombine i & j into p = p_i + p_j;
if the smallest is d_{iB}, declare i as a final jet;
and so on, until no particles are left

Cacciari, Salam, Soyez ‘08



“Cone” pattern with the soft radiation collected around hard towers

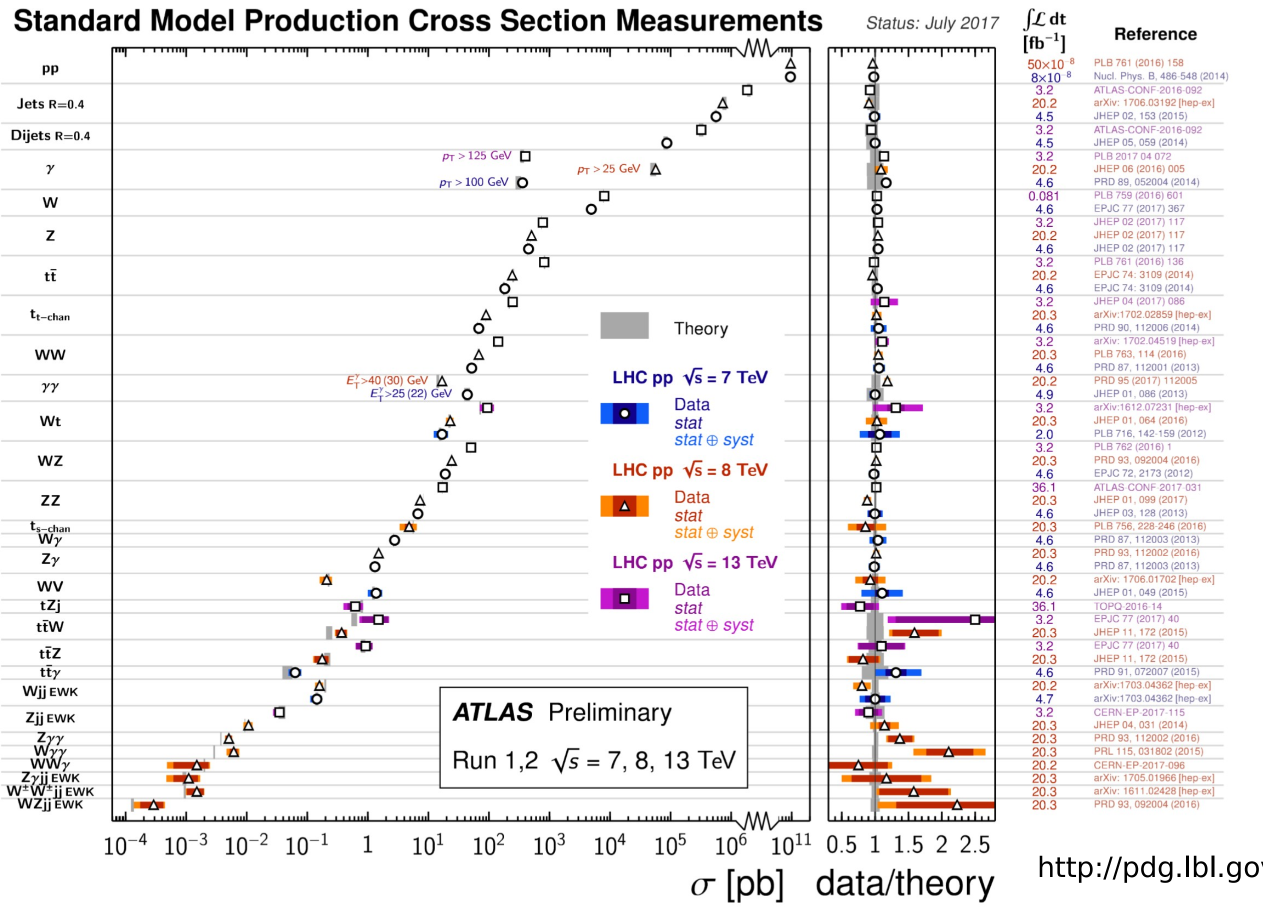
On the *experimental* definition of cross section

Number of expected event in time T

$$\sigma = \frac{N_{\rm ev}(T)}{\int_T \mathcal{L}(t)\mathrm{d}t}$$

Cross section = "probability" of a particular process

Luminosity = all the rest (collision frequency, bunch size, beam profile, ecc.)



http://pdg.lbl.gov/

On the *theory* definition of cross section

Partonic cross section for production n particles in the final state:

$$\sigma(2 \to n) = \frac{1}{\mathcal{F}} \int_{\Omega} |M_n|^2 \mathrm{d}\phi_n$$

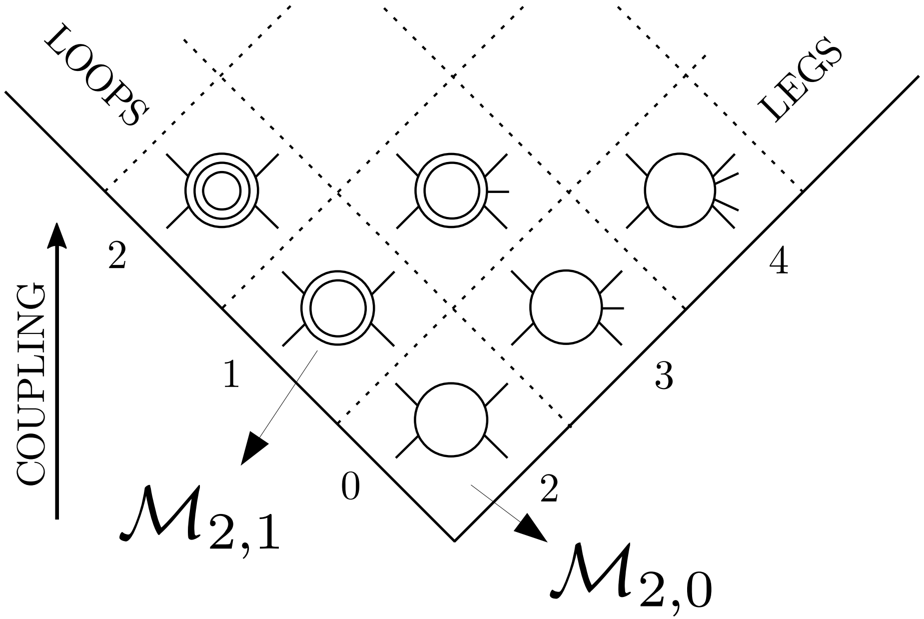
Integration over the final state phase space of n partons

Flux = function of initial state momenta

Amplitude squared = obtained from calculation of Feynman diagrams

Example: $e^+e^- \to q\bar{q}$

$$|M_2|^2 = \mathcal{M}_{2,0} + \mathcal{M}_{2,1}\alpha_s + \ldots$$



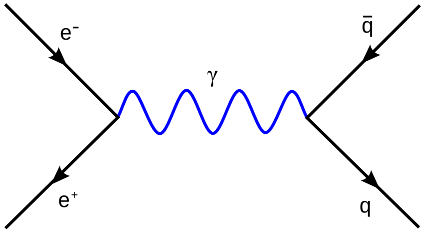
From partons to jets

A jet can contain more partons! e. g. a couple of partons close in angle (< R)

Leading order (LO) — Next-to-leading order (NLO)

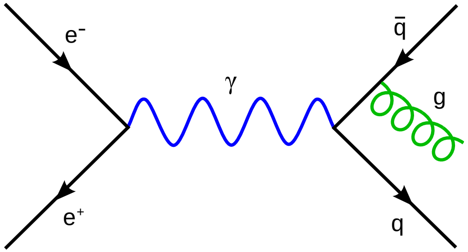
$$\sigma(e^+e^- \to 2 \text{ jets}) = c_0\alpha_s^0 + (c_{1,R} + c_{1,V})\alpha_s^1 + \ldots$$

$\propto \mathcal{M}_{2,0}$



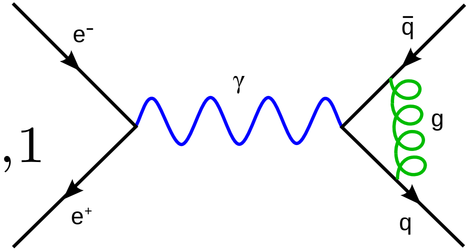
always 2 jets

$\propto \mathcal{M}_{3,0}$



Real term
2 or 3 jets

$\propto \mathcal{M}_{2,1}$

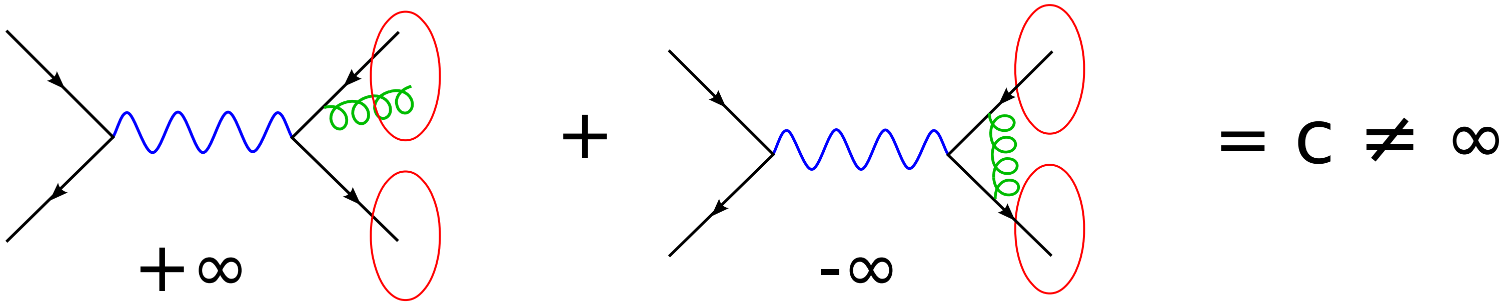


Virtual term
always 2 jets

IRC safety

A parton (2→2, 2→3, ecc.) cross section beyond LO is **infinite**!

BUT if jets are enough inclusive to allow the infinities to "sum up"...



Configuration when particles become SOFT or COLLINEAR (+∞)
same number of jets as the virtual (-∞)
=
INFRARED and COLLINEAR (IRC) SAFETY

Where do the infinities come from?

$$e^+e^- \to n \text{ partons with momenta } p_1, \ldots, p_n; n \text{ is a gluon}$$

When the gluon becomes:
- **soft** i.e. energy → 0
- **collinear** to another parton i.e. angle → 0

$$\lim_{\theta_{in} \to 0, E_n \to 0} \mathrm{d}\Phi_n |M_n^2(p_1, \ldots, p_n)| =$$

$$\mathrm{d}\Phi_{n-1} |M_{n-1}^2(p_1, \ldots, p_{n-1})| \quad \frac{\alpha_s C}{\pi} \frac{\mathrm{d}\theta_{in}^2}{\theta_{in}^2} \frac{\mathrm{d}E_n}{E_n}$$

"Single jet inclusive cross section"

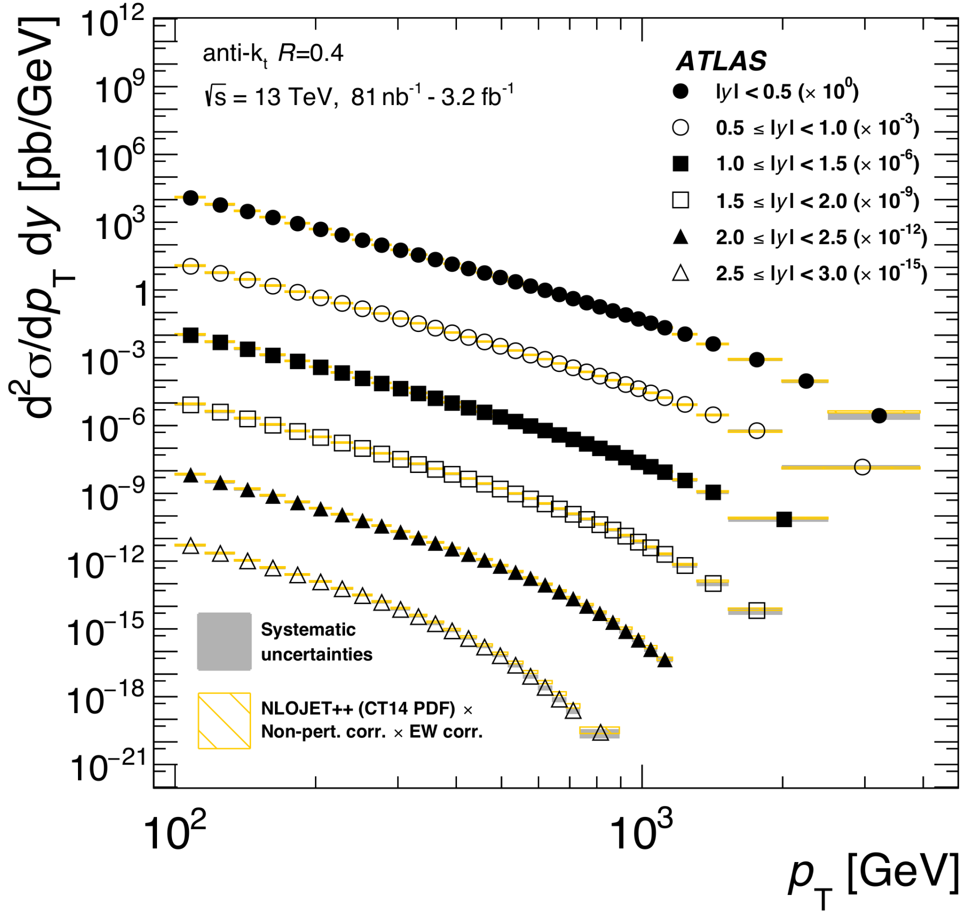
For a given event: all the jets

for each jet we plot pt (y)

A well known observable,
both from th. and exp. side.

NLO: Ellis, Kunszt, Soper (1992);
Giele, Glover, Kosower (1994); Nagy (2002)
NNLO: Currie, Glover, Pires (2017)

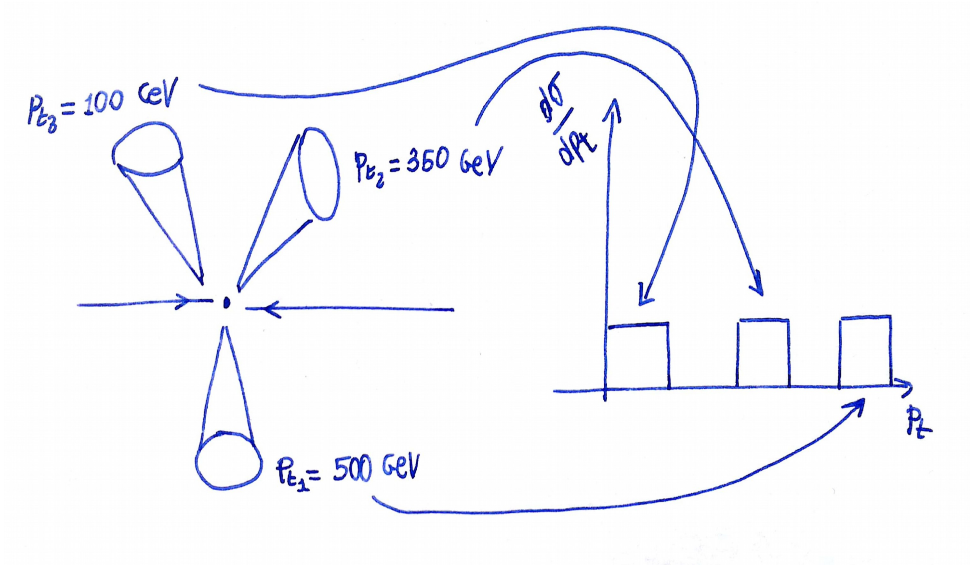
See e. g. ATLAS (CERN-EP/2017-157)
or CMS (CERN-EP/2016-104)



Main feature of this observable

Example: an event with 3 jets will fill the histogram three times!

→ This definition is **not unitary**!
i.e.

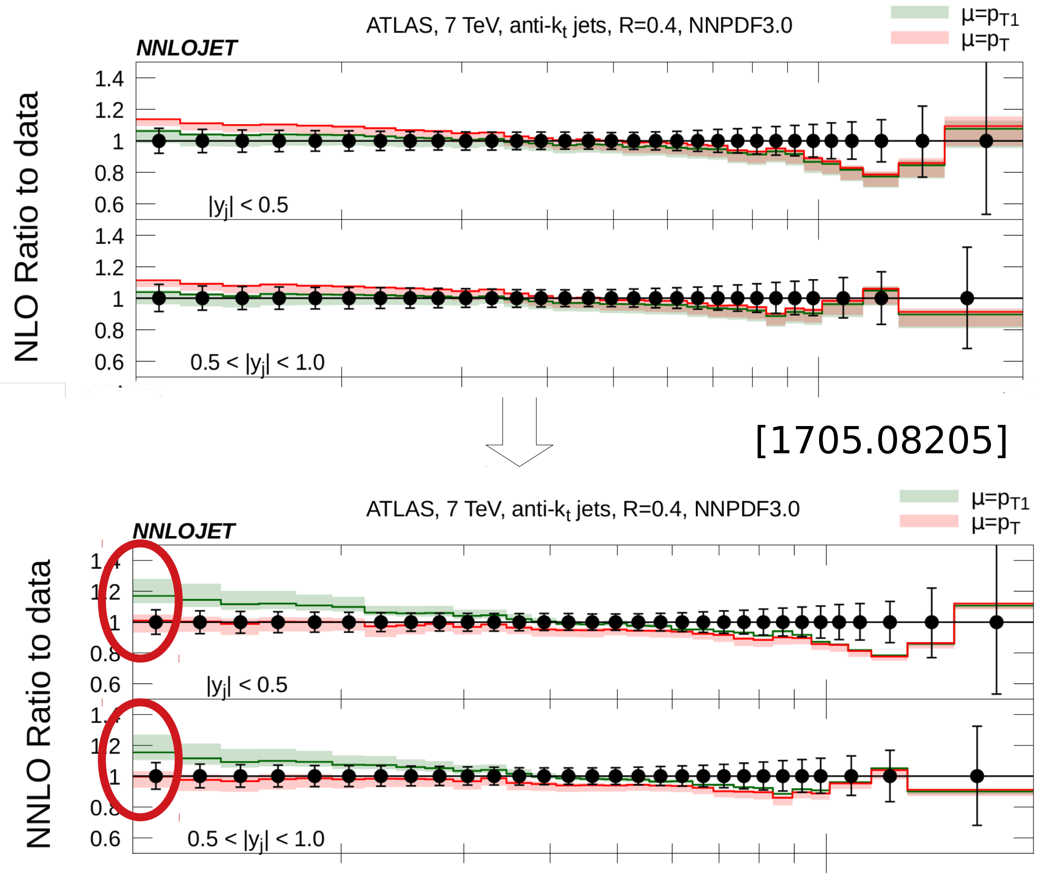


$$\sigma \neq \frac{N_{\rm ev}(T)}{\int_T \mathcal{L}(t)\mathrm{d}t} \quad \sigma_{3\mathrm{j}} = \frac{3N_{\rm ev}(T)}{\int_T \mathcal{L}(t)\mathrm{d}t}$$

$$\frac{\mathrm{d}\sigma_{\rm SJI}}{\mathrm{d}p_t} = \frac{\mathrm{d}\sigma_{1\mathrm{j}}}{\mathrm{d}p_t} + 2\frac{\mathrm{d}\sigma_{2\mathrm{j}}}{\mathrm{d}p_t} + 3\frac{\mathrm{d}\sigma_{3\mathrm{j}}}{\mathrm{d}p_t} + \dots$$

"three times the probability of having a jet in the event with a given pt ..."

What is the problem?



[1705.08205]

LHC precision era
→ now th error > exp error!
→ accuracy of predictions necessary
→ next order corrections needed

Why *scale variation*?
A "feeling" of theory uncertainty

Order increases
→ scale dependence decreases
= towards a perturbative "stability"

BUT in the recent NNLO calculation
this is not the case …

The "non-unitarity" consequence: logs mismatch

"What's happening at the boundaries?" Example with k getting soft

Potentially large logs with different signs

$$\frac{d\sigma_{3j}}{dp_{t,J}} = \sum_{i=1}^{3} \int dp_{t,i} dp_{t,j} dp_{t,k} \frac{d\sigma_{3j}}{dp_{t,i} dp_{t,j} dp_{t,k}} \delta(p_{t,J} - p_{t,i}) \supset \# \int_{p_{t,\mathrm{cut}}} \frac{dp_{t,k}}{p_{t,k}} \int dy_k d\varphi_k \mathcal{I}_{Jjk} \int dy_J d\varphi_J \mathcal{M}_{\mathrm{LO}}(p_J, p_J)$$

$$\frac{d\sigma_{2j}}{dp_{t,J}} = \sum_{i=1}^{2} \int dp_{t,i} dp_{t,j} \frac{d\sigma_{2j}}{dp_{t,i} dp_{t,j}} \delta(p_{t,J} - p_{t,i}) \supset \# \int^{p_{t,\mathrm{cut}}} \frac{dp_{t,k}}{p_{t,k}} \int dy_k d\varphi_k \mathcal{I}_{Jjk} \int dy_J d\varphi_J \mathcal{M}_{\mathrm{LO}}(p_J, p_J)$$

virtual

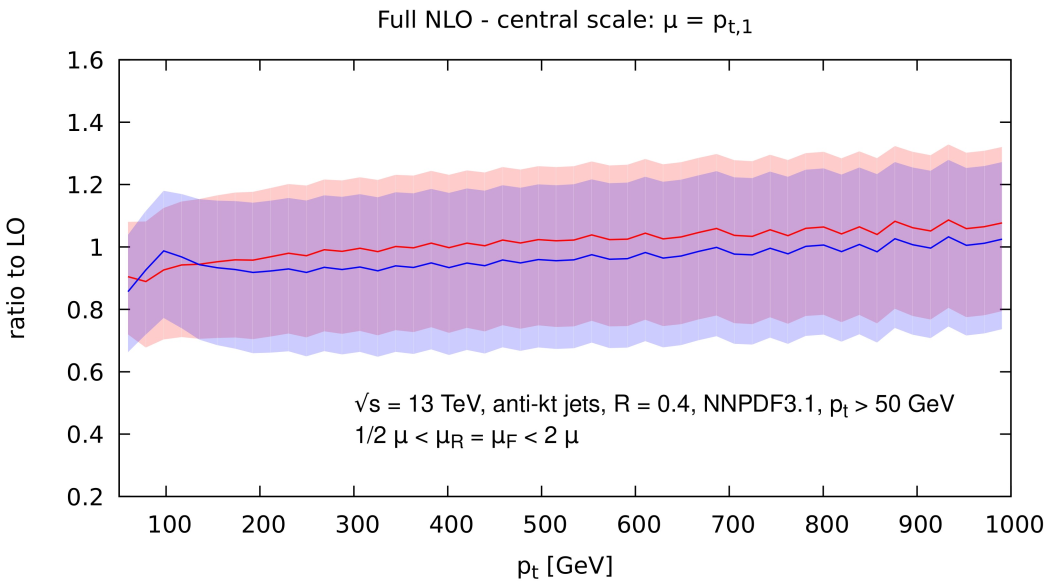
"Count" how many IRC logs appear in the 2 jets xs and in the 3 jets xs
Some of them will cancel, but other not

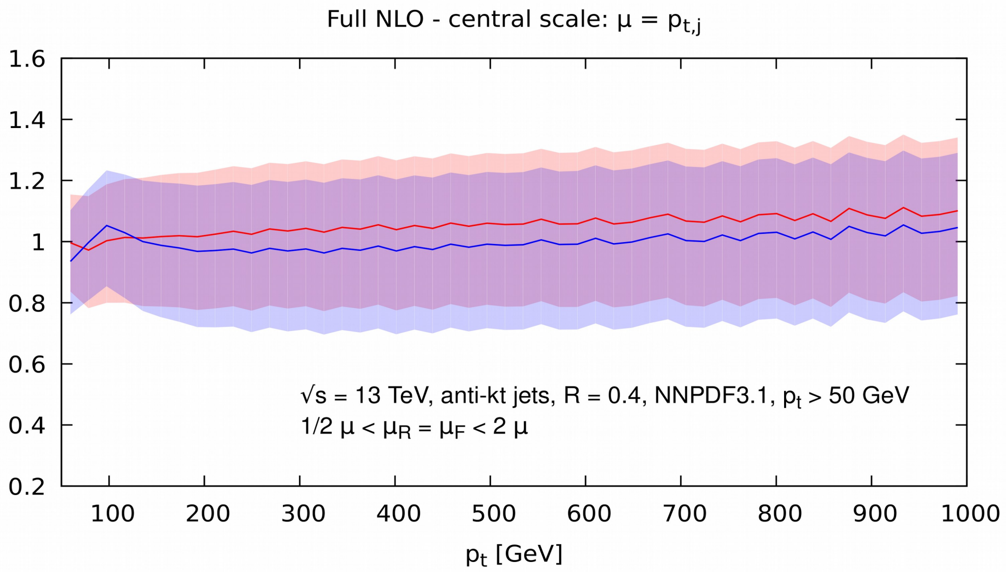
New ideas

PRELIMINARY

If we change definition of the observable maybe the perturbative stability of the calculation can be improved…

$$\frac{d\sigma_{\rm nj}}{dp_{t,J}} = \sum_{i=1}^{n} \int dp_{t,i} dp_{t,j} dp_{t,k} \ldots \frac{d\sigma_{\rm nj}}{dp_{t,i} dp_{t,j} dp_{t,k} \ldots} \delta(p_{t,J} - p_{t,i}) \frac{p_{t,i}}{p_{t,i} + p_{t,j} + p_{t,k} + \ldots}$$





Some final technical details

What we did during this first year:

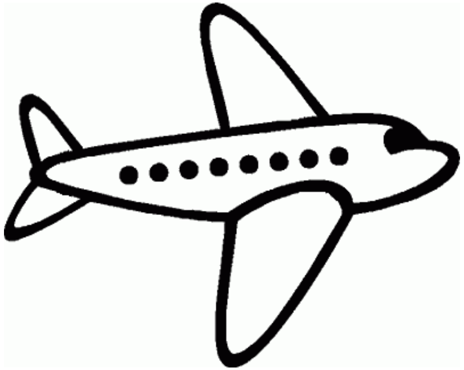
- analytic equation at NLO in the small R approximation when we can use the limits of the matrix element shown before (the full calculation is extremely difficult, can be done only numerically)

- we have tried different definitions in this approximated framework, by keeping results analytically when possible, otherwise doing integration numerically

- we have checked our predictions against a full numerical code, using for example NLOJet++ (Nagy 2001, 2003)

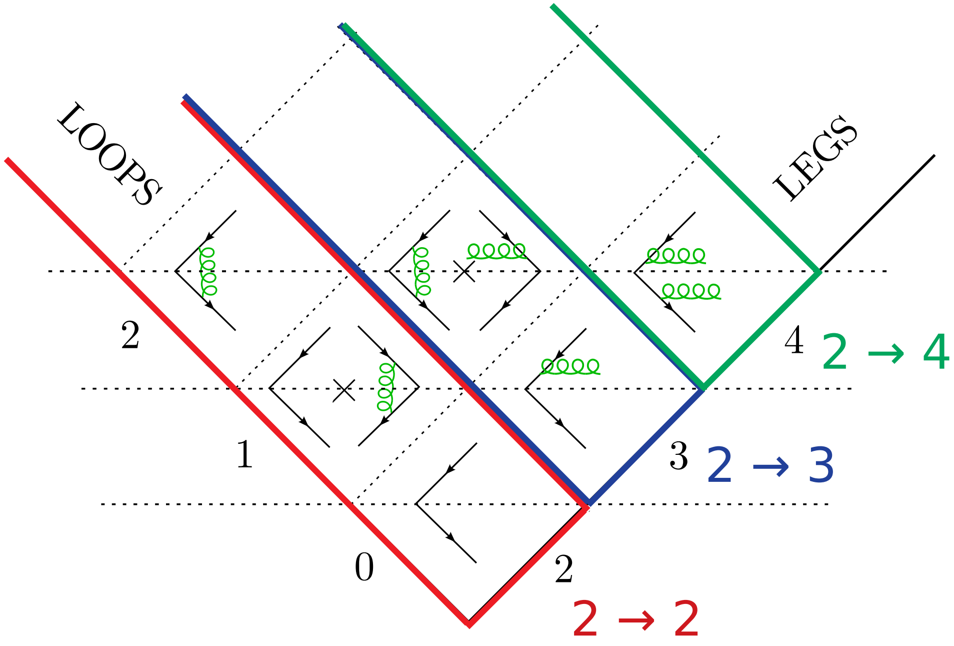
… result are going to appear very soon!

Thanks Milan, and see you very soon!

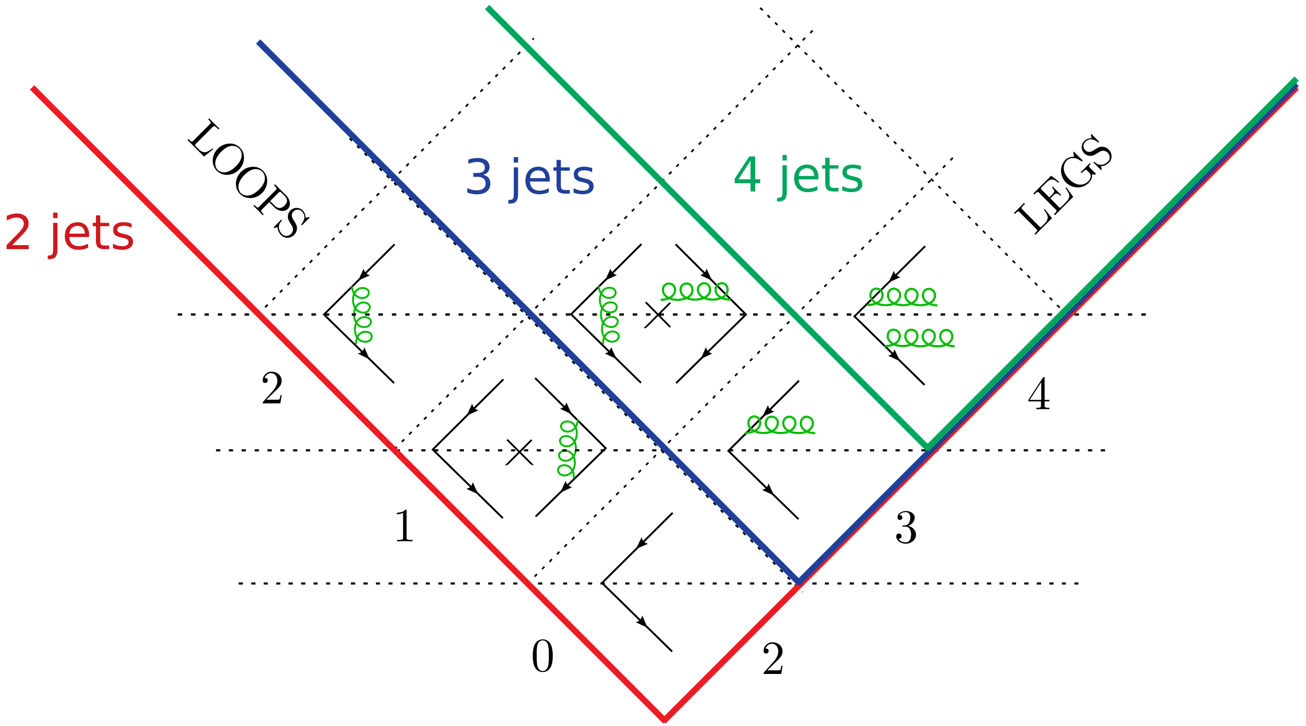


Backup slides

Partonic cross section



Jet cross section



Order by order, we need to ask for the kinematical configuration of the final state partons which give the required number of jets.

The “non-unitarity” consequence: logs mismatch

Our idea in a nutshell:

$$\sigma_{2\mathrm{j}} \supset \int_0^1 \frac{\mathrm{d}\theta^2}{\theta^2} \left(\Theta(\theta^2 < R^2) - 1\right) = -\int_{R^2}^1 \frac{\mathrm{d}\theta^2}{\theta^2} = -\log\left(\frac{1}{R^2}\right)$$

$$\sigma_{3\mathrm{j}} \supset \int_0^1 \frac{\mathrm{d}\theta^2}{\theta^2} \Theta(\theta^2 > R^2) = +\int_{R^2}^1 \frac{\mathrm{d}\theta^2}{\theta^2} = +\log\left(\frac{1}{R^2}\right)$$

We need to “count” how many logs appear in the 2 jets cross section and in the 3 jets cross section and to combine them properly, in order for these logs to cancel (or at least to be attenuated by some coefficient)