Pulsar glitches and neutron star masses

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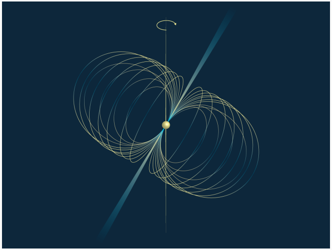
Neutron stars and pulsars

Extreme features of neutron stars:

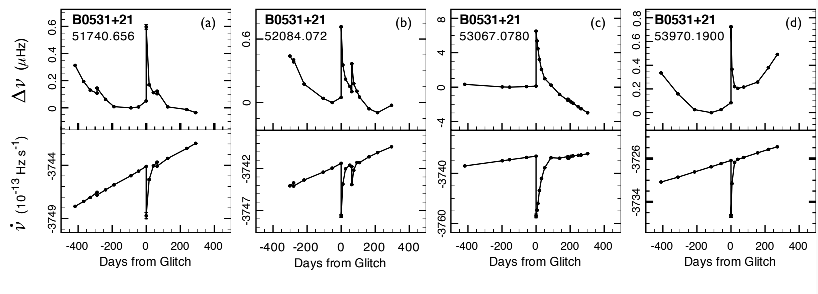
- R $\approx$ 10 km
- M $\approx 1 - 2\, M_{\odot}$
- $\rho_c \gtrsim 10^{14}$ g / cm^3
- $P \approx 10^{-3} - 10^{1}$ s
- $B \approx 10^{8} - 10^{15}$ G



Lighthouse model:

- Magnetic field axis misaligned with respect to the rotational axis
- Conversion of rotational energy into electromagnetic radiation
- Pulse frequency = rotational frequency

Pulsar glitches



Four glitches of the Crab pulsar, Espinoza et al. (2011)

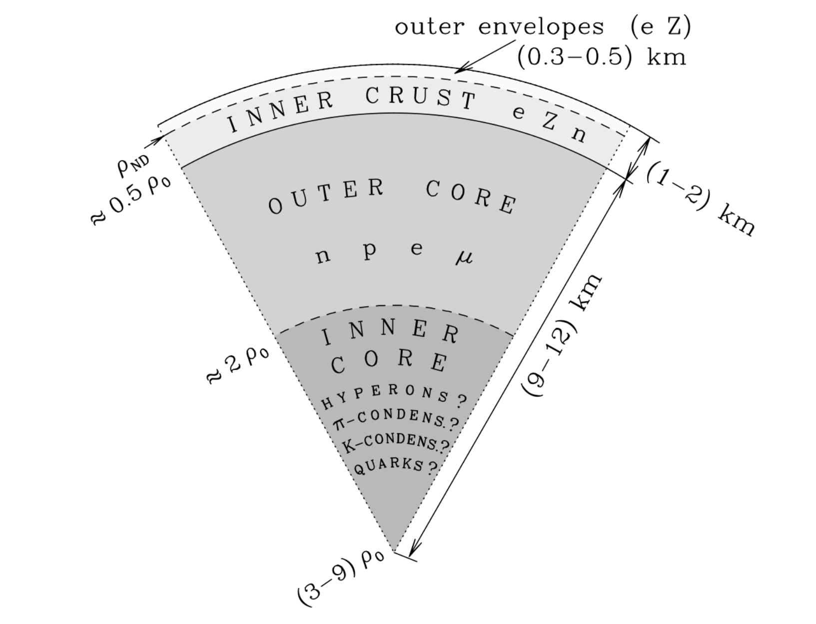
Two components: a normal charged one (visible) and a superfluid one.

Long recoveries $\Rightarrow$ Effect due to superfluid components.

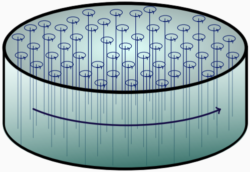
Diverse phenomenology:

- Mostly radiopulsars, but also magnetars and millisecond.
- Periodic glitchers vs single glitchers.
- Similar size vs. different size.

Neutron star structure

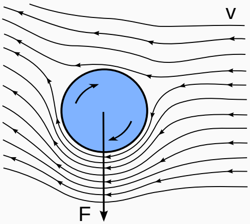


Glitch mechanism

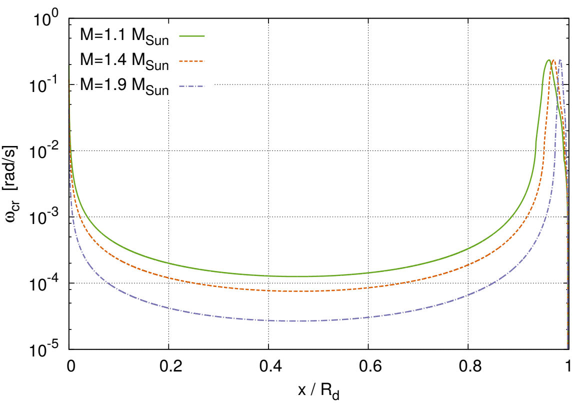


- The normal component loses angular momentum.
- The superfluid rotates by forming an array of quantised vortices.
- These vortices may pin to impurities of the crust.

The relative motion between the vortices and the superfluid itself causes a Magnus force. If the Magnus force is strong enough to overcome the pinning force, vortices detach and a glitch occurs.



Critical lag



$$\int_{\text{vortex}} f_P \, \mathrm{d}l = \int_{\text{vortex}} f_M \, \mathrm{d}l$$

Critical lag for vortices extended to the whole star (Antonelli & Pizzochero, 2017)

Maximal glitch

Angular momentum of the star:

$$L = I\Omega_p + \Delta L[\Omega_{np}]$$

Time derivative of the angular momentum:

$$I\dot{\Omega}_p + \Delta L[\dot{\Omega}_{np}] = -I|\dot{\Omega}_\infty|$$

Integration between a time before and one after the glitch:

$$\Rightarrow \Delta\Omega_{\rm max}(t) = \frac{\Delta L[\Omega_{np}(t)]}{I}$$

In the Newtonian framework:

$$\Delta L[\Omega_{np}] = \int \mathrm{d}^3x\, x^2 \rho_n(r)\, \Omega_{np}$$

Lag as a function of time

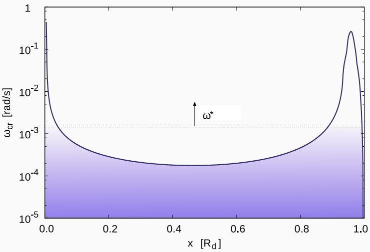
Our aim is that of calculating the time dependence of the lag.

The equations that allow us to calculate this value are complicated and they depend on the rotational parameters of the star (see Antonelli & Pizzochero 2017).

We can use a simplified (and unified) prescription that allow us to study the quantitative trend of the lag:

$$\Omega_{vp}(x, \omega^*) = \min\,[\Omega_{vp}^{\rm cr}, \omega^*],$$

where $\Omega_{vp} \equiv (1-\varepsilon_n)\Omega_{np}$ and $\omega^* \equiv |\dot{\Omega}_\infty| t$.



Pulsar activity and ω^*_{act}

The absolute pulsar activity is defined as:

$$\mathcal{A}_a = \frac{1}{T_{\mathrm{obs}}} \sum_{i=1}^{N} \Delta\Omega_i.$$

We can also define a timescale for a pulsar that defines the mean waiting time for a fictitious pulsar that exhibits only glitches of size $\Delta\Omega_{obs}$ and has activity $\mathcal{A}_a$:

$$t_{\mathrm{act}} = \frac{\Delta\Omega_{\mathrm{obs}}}{\mathcal{A}_a}.$$

Finally, we can find the nominal lag associated to this timescale:

$$\omega^*_{\mathrm{act}} = t_{\mathrm{act}} |\dot{\Omega}_\infty|.$$

Mass estimate M_{act}

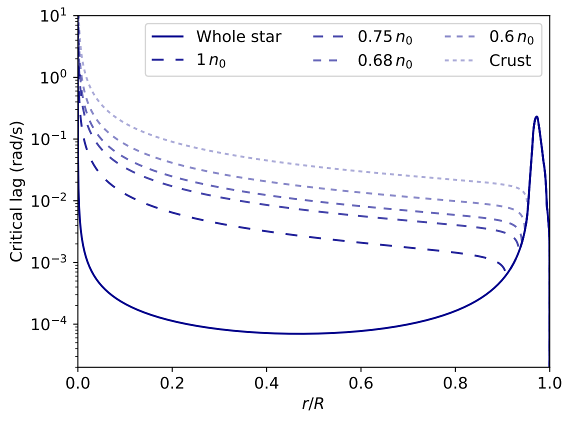
The maximal glitch $\Delta\Omega(\omega^*, M)$ depends only on the mass once the microphysical parameters are fixed.

Measuring a maximum glitch $\Delta\Omega_{\mathrm{obs}}$ and calculating the activity, we can estimate a mass for the pulsar inverting the relation:

$$\Delta\Omega(\omega^*_{\mathrm{act}}, M_{\mathrm{act}}) = \Delta\Omega_{\mathrm{obs}}.$$

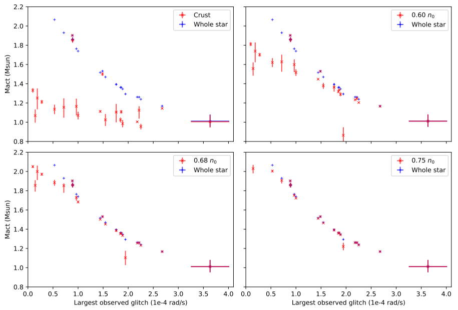
This mass estimate *depends* on the extension of the superfluid reservoir.

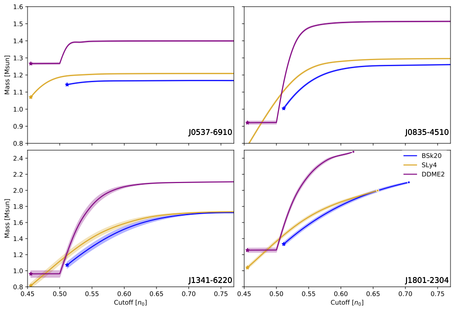
Critical lag



The critical lag depends on the cutoff considered.

Mass dependence on cuts





Montoli, Antonelli & Pizzochero, `arXiv:1809.07834`

Bayesian two-sample test: Antonelli, Gherardi, Montoli & Pizzochero, work in progress.

Conclusions

- The mass estimate M_{act} depends on the extension of the superfluid region.
- M_{act} is generally lower with a smaller superfluid reservoir.
- Like in other models (Andersson et al. 2012, Chamel 2013, Delsate et al. 2016), also here there is too little angular momentum stored in the crust for glitches.
- A little bit more of superfluid reservoir is enough to describe glitches.

Open question: Where actually resides the superfluid involved in the glitch?

Thank you!

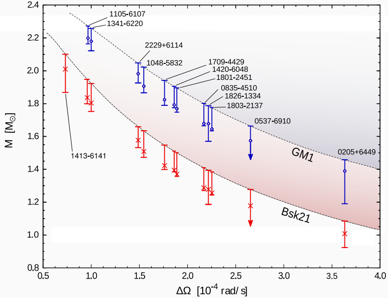
Mass upper limit $M_{\max}$

Maximum glitch = maximal glitch in the case of the critical lag.

$$\Delta\Omega_{\max} = \frac{\pi^2}{I\kappa}\int_0^{R_d} \mathrm{d}r\, r^3\, f_P(r).$$

As for $M_{\rm act}$, we can find a mass corresponding to the maximum glitch, $M_{\max}$. This is analytically *independent* on the extent of the superfluid region - as long as this extends at least in the pinning region - and entrainment paramters.

Already discussed in Pizzochero, Antonelli, Haskell & Seveso (2017) for the Newtonian case.



Slow rotation approximation

$$\frac{R^3\Omega^2}{GM} \ll 1$$

Slow rotation condition, Hartle (1967)

Extreme cases (millisecond pulsars, $M = 1.4M_\odot$, $R = 10km$):

- J1748-2446ad, $\Omega = 4501$ rad s^{-1} (not seen glitching):

$$R^3\Omega^2/(GM) \approx 0.11$$

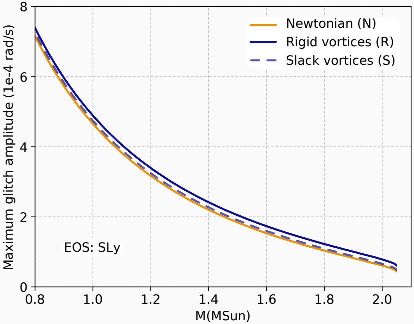
- J1824-2452A, $\Omega = 2057$ rad s^{-1} (seen glitching, Cognard & Backer 2004):

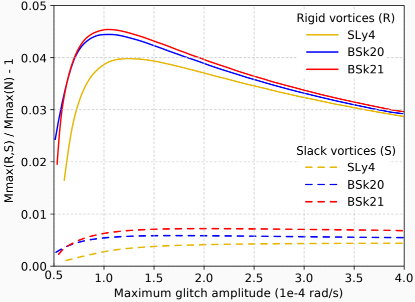
$$R^3\Omega^2/(GM) \approx 0.023$$

Mass upper limit $M_{\max}$, relativistic corrections

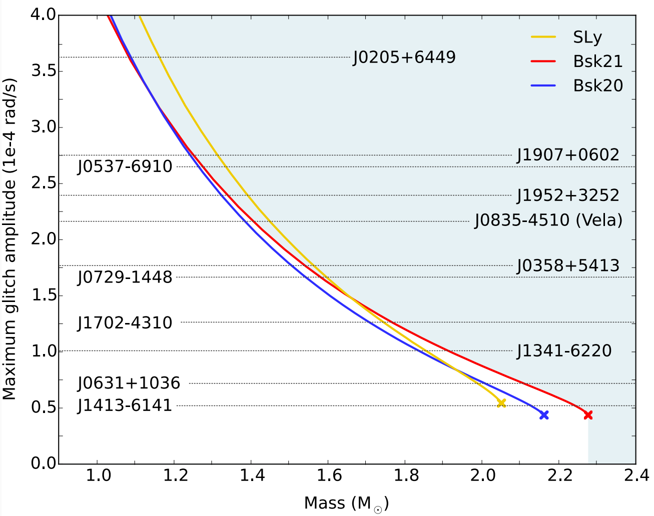
The calculation of the maximum glitch has been remade in the slow rotation framework in Antonelli, Montoli & Pizzochero (2018).

The dependence of $M_{\max}$ on relativistic corrections is weak.

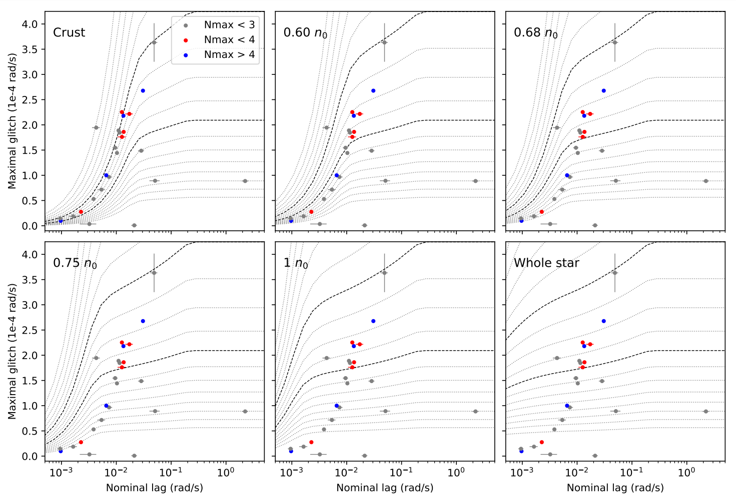




Maximum glitch amplitude



Maximal glitch



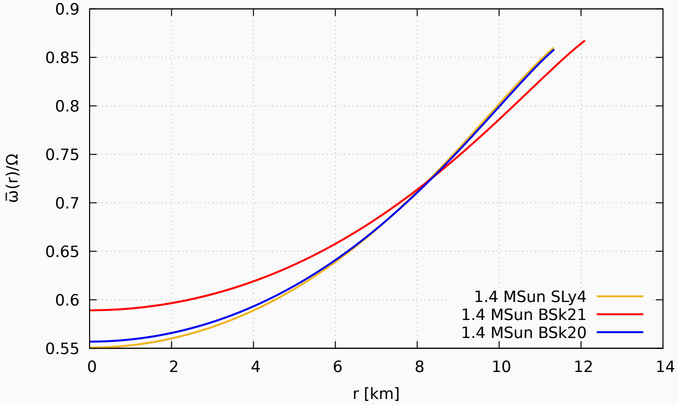
Pulsars have been chosen with $N_{\rm gl} > 2$, $N_{\rm max} > 1.1$, $T_{\rm obs}\,|\dot{\Omega}_\infty| > 10^{-3}$ rad/s.

Metric of a slowly rotating star

$$\mathrm{d}s^2 = -e^{2\Phi(r)}\mathrm{d}t^2 + e^{2\Lambda(r)}\mathrm{d}r^2 + r^2\left[\mathrm{d}\vartheta^2 + \sin^2\vartheta(\mathrm{d}\varphi - \omega(r)\mathrm{d}t)^2\right]$$

The radial profiles are still given by the TOV equations + EoS. $\omega(r)$ is given by an additional differential equation.

Drag of the local inertial frames



The relevant quantity is $\overline{\omega}(r) = \Omega - \omega(r)$.

Due to the linearity of the equation giving $\omega(r)$, $\overline{\omega}(r)/\Omega$ does not depend on Ω.

Moments of inertia

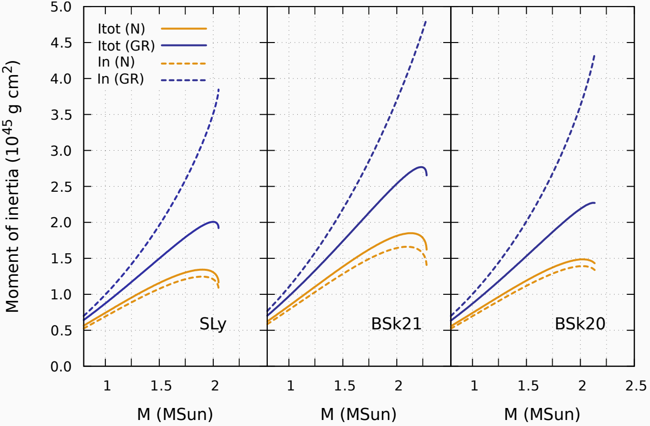
Total moment of inertia:

$$I = \frac{8\pi}{3} \int_0^R \mathrm{d}r \; r^4 e^{-\Phi(r)+\Lambda(r)} \left(\rho(r) + \frac{P(r)}{c^2} \right) \frac{\overline{\omega}(r)}{\Omega}$$

Moment of inertia of the superfluid component:

$$I_v = \frac{8\pi}{3} \int_0^R \mathrm{d}r \; r^4 e^{-\Phi(r)+\Lambda(r)} x_n(r) \left(\rho(r) + \frac{P(r)}{c^2} \right) \frac{m_n}{m_n^*(r)}$$

Moments of inertia



- The total moment of inertia varies up to 50%.
- In GR the moment of inertia of the superfluid reservoir exceeds the total one.