N=2 Supergravity in Anti-de Sitter Spacetimes

Matteo Azzola

10 October 2017

Outline

- Overview of the forces of nature and the need of a quantum theory of gravity
- String theory as a possible solution
- Supergravity and Compactification
- AdS/CFT correspondence
- N=2 d=5 supergravity
- New solution with running scalars that generalize the previous of Maldacena-Nunez[1]

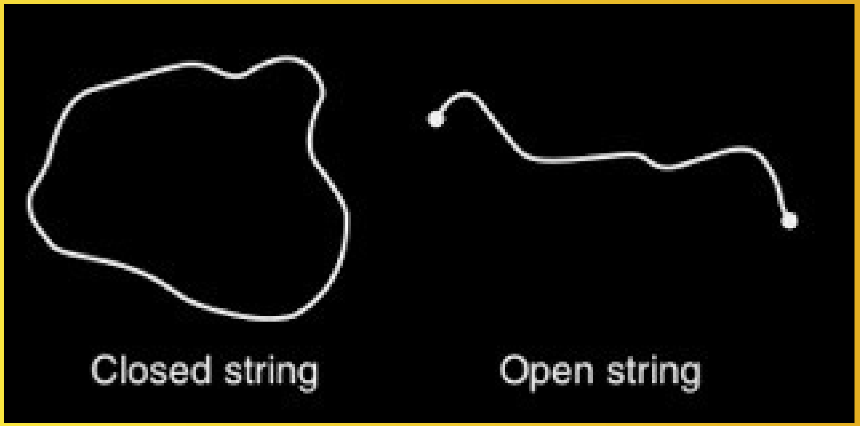
[1] J. M. Maldacena and C. Nunez, “Supergravity description of field theories on curved manifolds and a no go theorem,” Int. J. Mod. Phys. A 16 (2001) 822 [hep-th/0007018].

The four forces of nature

- Electromagnetic, weak and strong interaction are contained in the Standard Model (SM) of particle physics.
  - The SM is a quantum field theory that describes how particles propagate and interact with each other.
  - The whole theory is renormalizable, i.e. there is a finite number of free parameters in order to make finite the theory, at any order in the perturbation theory.
- Where is gravity ??
  - Unfortunately, GR is non-renormalizable. It cannot be introduced in the framework of SM.
- Who cares about renormalization! Is it that important?
  - Despite renormalization is a reductionist principle, it worked as in the Fermi theory of weak interaction.
  - Non-renormalization is in general a signal that new physics must appear at higher energy.

String Theory

- String theory is a theoretical framework that describes 1D objects, called strings, how they propagate throught the spacetime and how they interact with each other.



- Like the musical notes comes from excitation modes of a guitar string, point-like particles are given by different vibrational states of the string.
- Firstly introduced in the ‘60s, string theory was formulated as a first approach to the strong interaction.
- But besides gluons, the particle content of the theory was too rich.
- In 1974 Schwarz and Scherk showed that some of these particles could be interpreted as gravitons.
- This was the first signal that string theory could represent a consistent theory of quantum gravity.

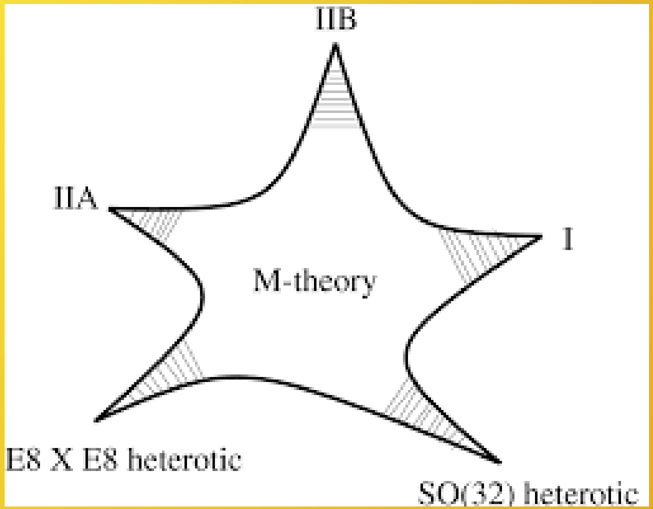
Superstring Theory

- The spectrum of the first string theory contained only Bosons.
- In order to have both bosons and fermions, supersymmetry must be introduced in string theory. The amount of supersymmetry is indicated by N.
- Lorentz invariance implies 10 spacetime dimensions.

5 superstring theories

Type-I   Type-IIA   Type-IIB   Het. SO(32)   Het. $E_8 \times E_8$

- These theories differ by the number of supersymmetries and the gauge group under which they transform.
- In 1995 Witten suggested that all these theories are different limiting cases of a single theory, all unified in the framework of M-theory.
- Dualities relate string theories among them.



Supergravity and Compactification

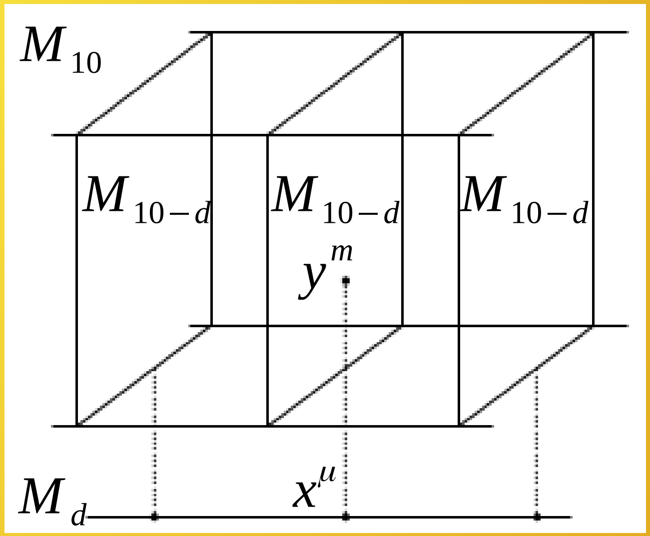
- Low-energy limit of a superstring theory, i.e. is an effective theory
- Massless spectrum

Weak-coupling $l_s^2 \sim \alpha' \rightarrow 0$ ——► String tension becomes large

$$M^2 \sim \frac{1}{\alpha'} \sum_{n=1}^{\infty} a_{-n}^i a_n^i$$ ⇒

- ✓ Massive states decouple
- ✓ Massless modes only

- A compactification scheme is needed in order to study a lower dimensional theory.
- Spacetime M_{10} is a fibration over M_d with fibers $M_{10\text{-d}}$

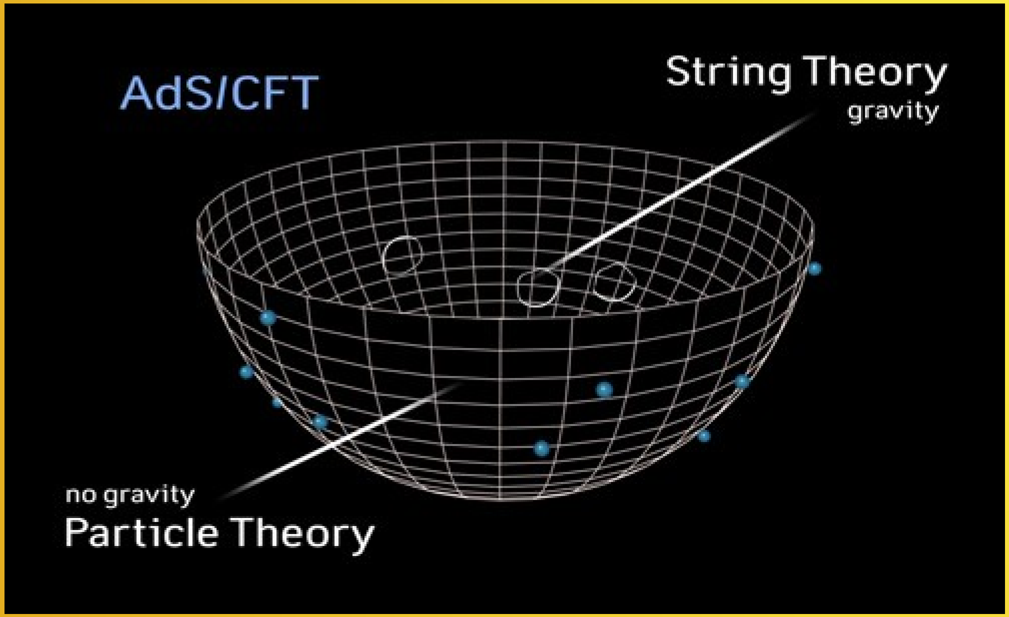


- In particular, the compactification of a 10d supergravity theory with the minimal amount of supersymmetry in d=4,5 is the N=2 supergravity.
- From now on we will focus only on the N=2 supergravity.

AdS/CFT correspondence

- First observed connection between a strongly coupled QFT and a gravitational theory

String Theory on AdS_{d+1} spacetime ⟺ DUAL ⟺ Conformal Field Theory on d-dimensional spacetime



- Field Theory lives on the conformal boundary of the anti-de Sitter spacetime.
- Bulk fields $\hat{h}(x, x_{d+1})$ become sources $h(x)$ for the CFT fields *O(x)*
- Interaction picture for CFT:

$$\int L_{CFT} + \int d^d x\, h(x) O(x)$$

Functional Generator W(h) is equivalent to the 5d Lagrangian

$$e^{W(h)} = \langle e^{\int hO} \rangle_{CFT} = e^{S_{AdS}(\hat{h})}$$

N=2 d=5 Supergravity

- Because the physical interesting QFT lives in 4 dimensions, we are interested in the dual 5d supergravities in AdS background.

- Bosonic part of the lagrangian in the FI U(1) gauge, coupled to n_v vector multiplets

$$e^{-1}\mathscr{L}=\frac{1}{2}R-\frac{1}{2}g_{ij}\partial_\mu\phi^i\partial^\mu\phi^j-\frac{1}{4}G_{IJ}F^I_{\mu\nu}F^{J\mu\nu}+\frac{e^{-1}}{48}C_{IJK}\epsilon^{\mu\nu\rho\sigma\lambda}F^I_{\mu\nu}F^J_{\rho\sigma}A^K_\lambda-g^2V$$

$$V=\frac{1}{18}g_I g_J\left(\frac{9}{2}g^{ij}\partial_i h^I\partial_j h^J-6h^I h^J\right)$$

where $h^I=h^I(\phi^i)$ are functions of the physical real scalar fields. The susy variations are

$$\delta\psi_\mu=\left(D_\mu+\frac{i}{8}h_I\left(\Gamma_\mu^{\nu\rho}-4\delta_\mu^\nu\Gamma^\rho\right)F^I_{\nu\rho}+\frac{1}{6\sqrt{2}}\Gamma_\mu h^I g_I\right)\epsilon$$

$$\delta\lambda_i=\left(\frac{3}{8}\Gamma^{\mu\nu}F^I_{\mu\nu}\partial_i h_I-\frac{i}{2}g_{ij}\Gamma^\mu\partial_\mu\phi^j+\frac{1}{2\sqrt{2}}g_I\partial_i h^I\right)\epsilon$$

Classical susy configuration

$$\delta\psi_\mu=0$$

$$\delta\lambda_i=0$$

Static configuration

- Ansatz for the metric and for the magnetic fluxes

$$ds^2 = e^{2V}(-dt^2 + dz^2) + e^{2W}(du^2 + d\Omega_k^2) \qquad F^I_{\theta\phi} = kq^I F_k(\theta) \qquad F_k(\theta) = \begin{cases} \sin(\theta), k=1 \\ \sinh(\theta), k=-1 \end{cases}$$

- The warp factors are written in therms of the scalars

$$e^{2V} = (x^1 x^2 x^3)^{-\frac{1}{3}} e^{-g\int(x^1 x^2 x^3)du} \qquad e^{2W} = (x^1 x^2 x^3)^{\frac{2}{3}}$$

- Introducing the linear combination

$$y^1 = x^1 + x^2 - x^3$$
$$y^2 = x^1 - x^2 - x^3$$
$$y^3 = x^1 - x^2 + x^3$$

- And rewriting the charges in the parameters

$$Q^1 = -k(q^1 + q^2 - q^3)$$
$$Q^2 = -k(q^1 - q^2 - q^3)$$
$$Q^2 = -k(q^1 - q^2 + q^3)$$

- The supersymmetric flow equations for the scalars are

$$y^1{}' = y^2 y^3 + Q^1$$
$$y^2{}' = y^1 y^3 + Q^2$$
$$y^3{}' = y^1 y^2 + Q^3$$

Non-homogeneous version of Nahm equations

Generalizing Maldacena-Nunez

- A new solution can be found taking $Q^1 = Q^2 = 0$

$$ds^2=(x^1x^2x^3)^{-\frac{1}{3}}e^{g\int\frac{(x^1+x^2+x^3)}{y}du}(-dt^2+dz^2)+(x^1x^2x^3)^{\frac{2}{3}}\left(\frac{1}{(y^3)^2}dy^2+d\Omega_k^2\right)$$

- The solution of the Nahm system gives the scalar fields

$$x^1=\frac{1}{4}\left\{k_1e^{-gy}+k_2e^{gy}+\sqrt{k_1^2e^{-2gy}+k_2^2e^{2gy}+\frac{8ky}{g}}\right\}$$

$$x^2=\frac{1}{2}k_2e^{gy}$$

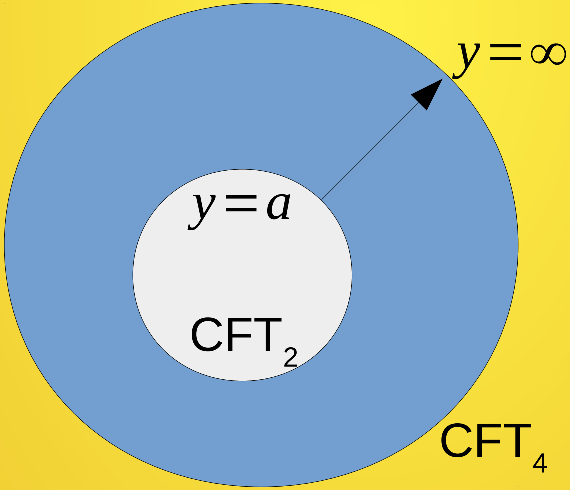
$$x^3=\frac{1}{4}\left\{-k_1e^{-gy}+k_2e^{gy}+\sqrt{k_1^2e^{-2gy}+k_2^2e^{2gy}+\frac{8ky}{g}}\right\}$$

Generalizing Maldacena-Nunez

- A new solution can be found taking $Q^1 = Q^2 = 0$

$$ds^2 = (x^1 x^2 x^3)^{-\frac{1}{3}} e^{g\int \frac{(x^1+x^2+x^3)}{y} du} (-dt^2 + dz^2) + (x^1 x^2 x^3)^{\frac{2}{3}} \left(\frac{1}{(y^3)^2} dy^2 + d\Omega_k^2 \right)$$

- This solution is an interpolation flow between AdS_5 and AdS_3 x H^2 , for y that goes from infinity to the horizon $y = a$



- The value $k_1 = 0$ corresponds to the limit in which $x^1 = x^3$. This truncation is the MN solution.

CFT dual picture

- The physical scalar fields are

$$\frac{2}{\sqrt{6}}\phi_1 = \log\left(\frac{x^2}{(x^1x^2x^3)^{\frac{1}{3}}}\right) \qquad \sqrt{2}\,\phi_2 = \log\left(\frac{x^3}{x^1}\right)$$

- Calculated on the conformal boundary of AdS

$$\frac{2}{\sqrt{6}}\phi_1 \sim 2Qye^{-2gy} \qquad \sqrt{2}\,\phi_2 \sim \frac{-k_1}{k_2}e^{-2gy}$$

- In the dual SCFT these are an expectation value of an operator and an insertion of dimension 2.

- The central charge of the 2d SCFT dual to the horizon configuration AdS_3 x H^2 is

$$c = \frac{6\pi(g-1)}{G_5}\frac{Q}{4g}$$ where g is the genus of the Riemann surface H^2

- Again, the truncation $k_1 = 0$ corresponds to the MN value.

Conclusion

- We have seen very brief introduction to superstrings
- We have given some basics of the AdS/CFT correspondence
- The N=2 d=5 supergravity model has been presented
- A new solution has been found, it generalizes that of MN and describes a flow across dimensions.