XXXI PhD cycle - 1st year Workshop
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Nonclassicality in Continuous Variables Quantum Systems

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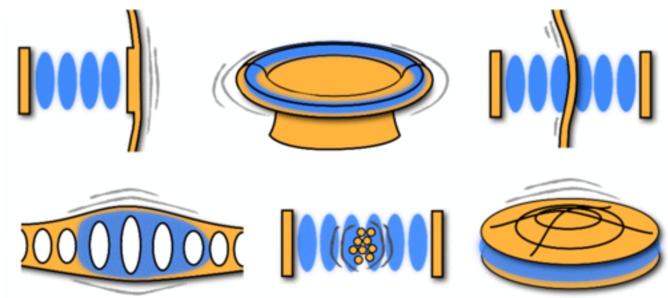
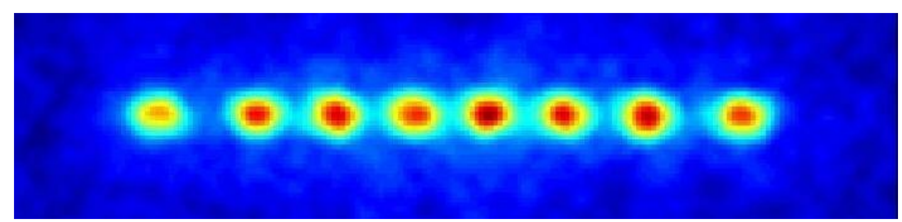
Continuous Variable Quantum Systems

- Quantum optical systems

- Optomechanical systems



- Trapped ions



- General bosonic degrees of freedom

Notation for Bosonic Single-Mode Continuous Variable Systems

- Bosonic field operators $$\left[\hat{a}, \hat{a}^\dagger\right] = 1$$
- Quadrature operators $$\hat{x}_\phi = \frac{1}{\sqrt{2}}\left(\hat{a}e^{-i\phi} + \hat{a}^\dagger e^{i\phi}\right)$$
  $$\hat{x}_0 = \hat{q} \quad \hat{x}_{\pi/2} = \hat{p}$$
- Coherent states $$|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle \quad \alpha \in \mathbb{C}$$
  $$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$$
- Discrete Fock basis $$\rho = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} \rho_{nm}|n\rangle\langle m|$$
- Continuous position basis $$\rho = \int \mathrm{d}x\mathrm{d}y\, \rho(x,y)|x\rangle\langle y|$$

Quantum Mechanics in Phase Space

- Single mode: 2D (classical) phase space
- Quantum mechanics can be entirely formulated in phase-space
- Quantum states represented as "quasi-distributions" in phase space (not unique!)

- **Wigner function**

$$W(q,p) = \frac{1}{2\pi} \int \mathrm{d}y \, \langle x + \frac{y}{2} | \rho | x - \frac{y}{2} \rangle e^{-iyp}$$

- Marginals are the quadrature distributions
- Hamiltonian linear or quadratic in q and p

  → **linear transformations** of the mode operators

  → **classical phase space dynamics** (Liouville equation)

What is nonclassicality?

- Many different notions!
- Might be **physical-context dependent** (e.g. light vs matter excitations)
- Multipartite systems → nonclassical correlations (entanglement, discord)
- Basis dependent notions → coherence (“classical” if diagonal in some basis)
- Focus: **single mode** systems (no subsystems) and

  phase-space based criteria & measures

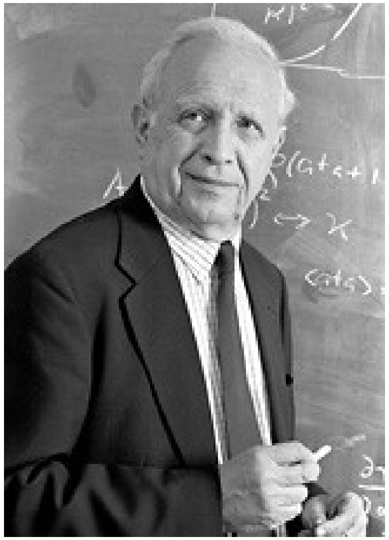
Glauber P-Nonclassicality

- P is a different quasi-probability distribution (related to the Wigner via Gaussian convolution)
- If P is positive & “well-behaved” ρ is a mixture of coherent states
- **Quantum optics**: states that show a phenomenology explained by Maxwell equations

$$\rho = \int \mathrm{d}^2\,\alpha P(\alpha)|\alpha\rangle\langle\alpha|$$

- **If P is NOT positive & “well-behaved”**

  → ρ is P-nonclassical



Roy Glauber
Nobel Prize 2005

Examples of Glauber P-Nonclassicality

P-classical

- Coherent states
- Thermals states
- Roughly speaking: quantum states of light implemented by **linear optics**

P-nonclassical

- Fock states $|n\rangle$
- Squeezed states

$$|r\rangle = e^{\frac{1}{2}\left(r^* \hat{a}^2 - r \hat{a}^{\dagger 2}\right)}|0\rangle$$

- "Cat" states

$$\propto |\alpha\rangle \pm |-\alpha\rangle$$

- **Nonlinear optical elements** needed

Non-Gaussian states

Gaussian states:

- Wigner function is a Gaussian
- Thermal or ground states of linear and quadratic Hamiltonians
- "Easily" created in quantum optics labs
- **Hudson's theorem:**

  The only W-classical **pure states** are **Gaussian** states

Non-Gaussian states:

- Used to improve some CV quantum-information protocols (e.g. teleportation, cloning)
- Usually more difficult to implement & control in the lab
- Also non-Gaussianity can be quantified [Genoni & Paris PRA 2008]

Negativity of the Wigner function

- The Wigner function is always well defined, but can have **negative values**
- Quantum circuits with initial states and quantum operations characterized by positive Wigner functions can be **classically efficiently simulated**

[Mari & Eisert, PRL 2012; Veitch et al. , NJP 2012]

- **If W is NOT positive**

  → ρ is W-nonclassical
- **Quantification** of W-nonclassicality via volume of negative part

[Kenfack & Życzkowski, J. Opt. B 2004]

Examples of W-nonclassicality

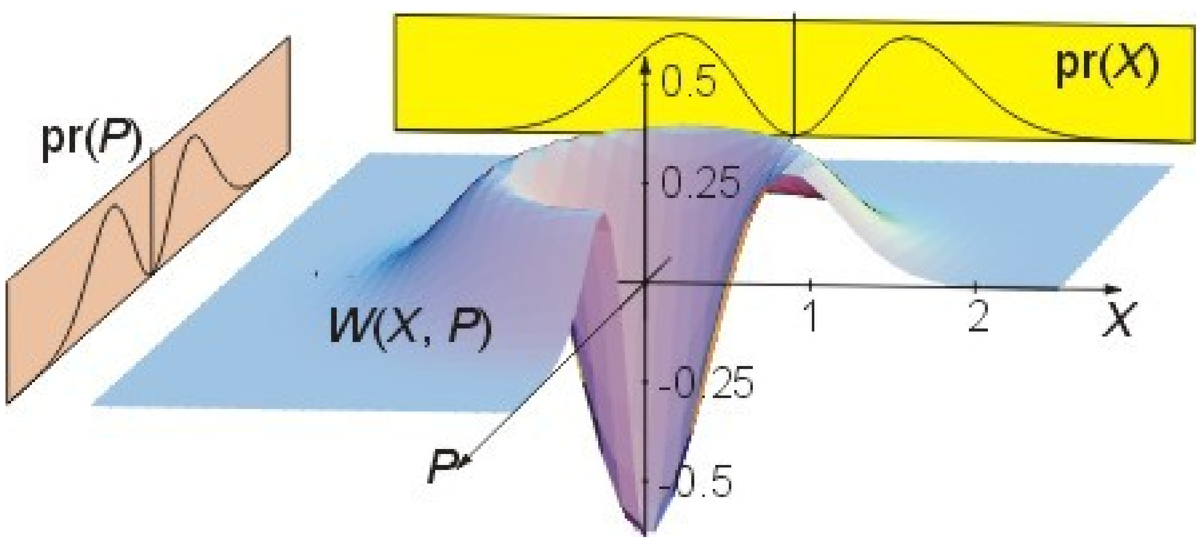
W-classical

- Every Gaussian states (including squeezed states)

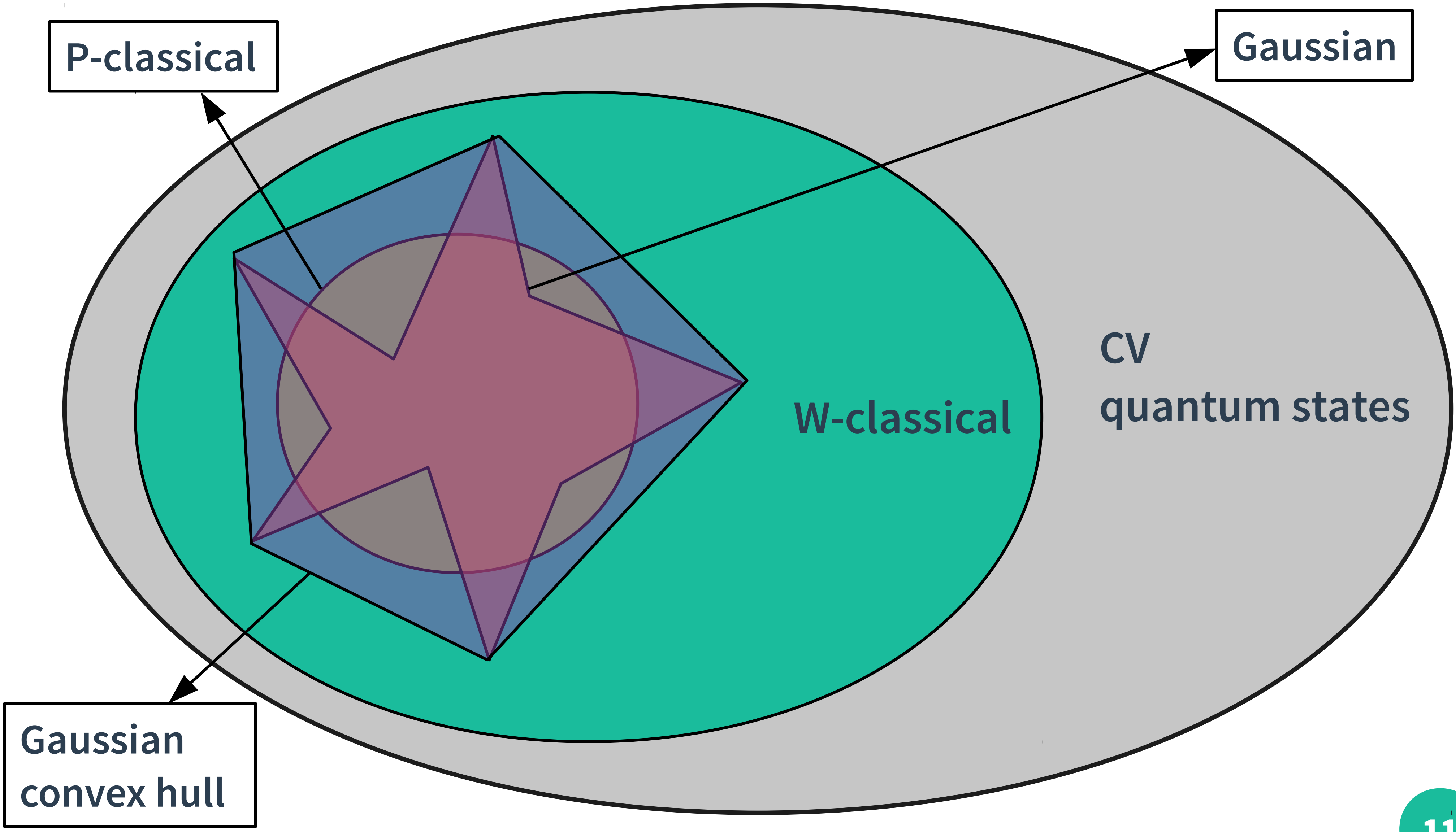
W-nonclassical

- Fock states
- “Cat” states

Wigner function of $|1\rangle$



“Zoology” of CV quantum states

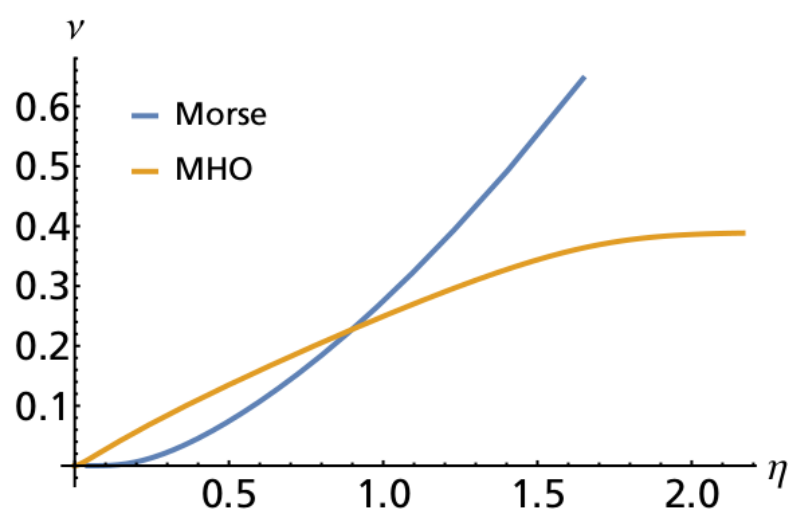


Present Research Lines

- Comparison & study of the interplay between different forms of nonclassicality
- Find “practical” applications where some form of nonclassicality brings an advantage
- Find the proper physical “resources” for different tasks

Comparison Between Different Forms of Nonclassicality

- **Monotony** between measures of non-Gaussianity & W-nonclassicality for **ground states of anharmonic oscillators** (not guaranteed by Hudson's theorem) [Albarelli et al. PRA 2016]



- Comparison between W-nonclassicality and **quantum probability backflow** [Albarelli, Guaita & Paris, IJQI 2016]

Metrological Application of non-Gaussian states

- "Practical" problem: **estimation of the loss** coefficient of a bosonic lossy channel (well studied problem)
- New: add a **nonlinear self-Kerr interaction** (i.e. change the channel) and use Gaussian input states
- **Improved estimation** (figure of merit: Quantum Fisher Information), relevant in some regimes of the parameters [Rossi, Albarelli & Paris, PRA 2016]

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = \gamma \left(\hat{a}\rho\hat{a}^\dagger - \frac{1}{2}\hat{a}^\dagger\hat{a}\rho - \frac{1}{2}\rho\hat{a}^\dagger\hat{a} \right)$$

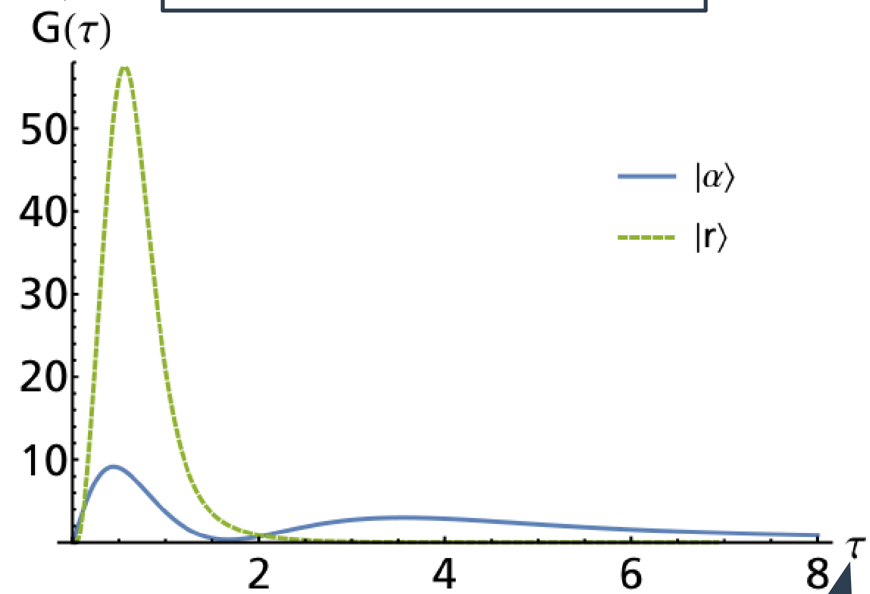
$$\hat{H}_K = \lambda \left(\hat{a}^\dagger \hat{a} \right)^2$$

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = i \left[\hat{H}_K, \rho \right] + \gamma \left(\hat{a}\rho\hat{a}^\dagger - \frac{1}{2}\hat{a}^\dagger\hat{a}\rho - \frac{1}{2}\rho\hat{a}^\dagger\hat{a} \right)$$

Metrological Application of non-Gaussian states

- A substantial improvement for "small-times"
- Kerr interactions brings the state outside the set of Gaussian states
- BUT not clear which particular "direction" of the Hilbert to explore for increased precision, i.e. the **physical resource** is not clear yet

Gain in the QFI w.r.t. the Gaussian case, in %



Rescaled time parameter

Future Research Lines

- Assess the role of W-nonclassicality in practical **CV quantum computation** schemes
- Possibility of defining a proper **resource theory** for some form of nonclassicality (recently done for squeezing [Idel, Lercher & Wolf, JPA 2016])
- Comparison between nonclassicality and "**complexity**" of phase-space distributions
- Extend previous analysis of ground states to **thermal states** of anharmonic oscillators

Detailed List of Cited References

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