From Strings to AdS_4-Black holes

by Marco Rabbiosi

13 October 2015

Why quantum theory of Gravity?

The four foundamental forces are described by

- Electromagnetic, weak and strong $\rightarrow$ ***QFT*** (Standard Model)
- Gravity $\rightarrow$ ***Differential Geometry*** (General Relativity)

- These are two very ***different mathematical frameworks***... ..but a common belief is the existance of a ***foundamental microscopic theory*** able to describe all together.
- ***Canonical quatization of GR fails*** because of problems with unitarity and renormalizability... ...however for a complete understanding of ***Black holes*** and ***Big Bang*** we need Quantum Gravity: they exibit a ***singular behaviour in GR,*** curvature scalars diverge.
- Characteristic lenght: $l_{ph} = \sqrt{\frac{\hbar G}{c^3}} \sim 10^{-35} m \rightarrow$ direct epxerimental data inaccessible...but on the theoretical side...

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Supersymmetry

- Enlargment of $ISO(3,1)$ $\rightarrow$***Coleman-Mandula No-go theorem***.

4d SuperPoincarè algebra $\mathfrak{s}_{\mathcal{N}} = \{P_\mu, J_{\mu\nu}, Q^i_\alpha = (Q^i_A, \bar{Q}_{\dot{A}i})^t, T_r\}$

$$\{Q^i_A, \bar{Q}_{\dot{B}j}\} = -2i(\sigma^\mu)_{A\dot{B}}\,\delta^i_j P_\mu\,, \quad \{Q^i_A, Q^j_B\} = \varepsilon_{AB}\left(U^{ij} + iV^{ij}\right),$$

$$[Q^i_A, T_r] = (U_r)^i{}_j\, Q^j_A \quad [Q^i_A, J_{\mu\nu}] = \frac{1}{2}(\sigma_{\mu\nu})_A{}^B\, Q^i_B$$

plus complex conjugate relations and the usual Poincaré algebra.

- The ***representations are*** particular ***multiplets*** of fields with *Fermionic d.o.f. = Bosonic d.o.f* on shell.
- Very ***strict conditions*** on a supersymmetric field theory (dimensions, interactions...).

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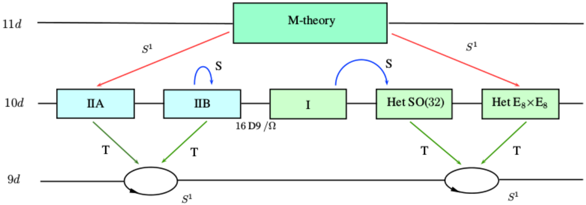
M-theory

- ***Gauging supersymmetry*** requires diffeomorphism invariance, ...the symmetry of GR!...we obtain ***Supergravity!***
- Moreover, the ***quantum spectrum of a string*** always has a massless spin 2 particle***...again Gravity!***

These ingredients lead to *M-theory*

- $d = 11$ supersymmetric theory of ***strings and branes***.
- Beyond an ordinary QFT: a ***net of duality between superstring theories*** that describe it in different limits.



- Interesting to study the ***low energy limit*** ($\alpha' \to 0$) of these theories, the ***massless part of the spectrum***.
- The ***effective field theory*** results to be a ***Supergravity*** theory in $d = 10, 11$!
- ***Compactifications*** means they admit solutions like $\mathscr{C}_s \times \mathscr{M}_{d-s}$.
- They ***link the different Supergravities*** and their deformations ***in all the dimensions***!
- In particular $\mathscr{N} = 2$ $d = 4$ ***gauged supergravity*** comes from compactifications on a flux background.
- ***We search black hole solutions*** in the e.o.m. of this theory...main motivations:

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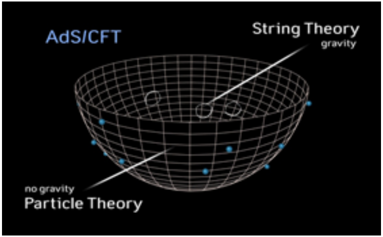

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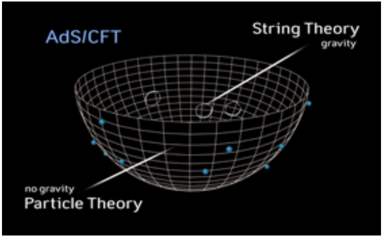
- ***Asintotically*** Ads_4 ***black hole*** solutions could be ***dual to some deformations of*** $ABJM$.
- Lifting to ***vacuas of M-theory***...$M2$-branes wrapping a Riemann surface.
- Study of ***microstates of black holes***.
- ***Integrability properties*** of classical GR in $d = 4$.



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- The ***bosonic part*** of the action (fermionic configuration to zero is a consistent truncation) ***reads***

$$\mathcal{L}=\sqrt{-g}\Big(\frac{R}{2}-h_{uv}\nabla_\mu q^u\nabla^\mu q^v-g_{i\bar{j}}\partial_\mu z^i\partial^\mu\bar{z}^{\bar{j}}+\frac{1}{4}I_{\Lambda\Sigma}H^{\Lambda\mu\nu}H^\Sigma_{\mu\nu}$$

$$+\frac{R_{\Lambda\Sigma}H^\Lambda_{\mu\nu}\varepsilon^{\mu\nu\rho\sigma}H^\Sigma_{\rho\sigma}}{8\sqrt{-g}}-(4h_{uv}k^u_\Lambda k^v_\Sigma X^\Lambda\bar{X}^\Sigma+(f_i^\Lambda g^{i\bar{j}}\bar{f}^\Sigma_{\bar{j}}-3X^\Lambda\bar{X}^\Sigma)P^x_\Lambda P^x_\Sigma)$$

$$-\frac{1}{4}\frac{\varepsilon^{\mu\nu\rho\sigma}}{\sqrt{-g}}\Theta^{\Lambda a}B_{\mu\nu a}\partial_\rho A_{\sigma\Lambda}+\frac{1}{32\sqrt{-g}}\Theta^{\Lambda a}\Theta^b_\Lambda\varepsilon^{\mu\nu\rho\sigma}B_{\mu\nu a}B_{\rho\sigma b}\Big),$$

- Where the ***covariant derivative*** of the hyperscalars is

$$\nabla_\mu q^u=\partial_\mu q^u+A^\Lambda_\mu k^u_\Lambda-A_{\Lambda\mu}k^{\Lambda u},$$

and $H^\Lambda_{\mu\nu}=F^\Lambda_{\mu\nu}+\frac{1}{2}\Theta^{\Lambda a}B_{\mu\nu a}$ are ***modified field strenghts*** (necessary for having also magnetically charged hyperscalars).

Ansatz

- We choose the ***more simple possible ansatz*** for the ***metric***

$$ds^2 = -e^{2U(r)}dt^2 + e^{-2U(r)}dr^2 + e^{2\psi(r)-2U(r)}d\Omega_l^2\,,$$

where

$$f_l(\theta) = \frac{1}{\sqrt{l}}\sin(\sqrt{l}\theta) = \begin{cases} \sin\theta\,, & l=1\,,\\ \sinh\theta\,, & l=-1\,,\end{cases}$$

and for the ***gauge fields***

$$A^\Lambda = A_t^\Lambda dt - lp^\Lambda f_l'(\theta)d\phi\,,\quad A_\Lambda = A_{\Lambda t}dt - lq_\Lambda f_l'(\theta)d\phi\,,$$

$$B^\Lambda = 2lp'^\Lambda f_l'(\theta)dr\wedge d\phi\,,\quad B_\Lambda = -2lq'_\Lambda f_l'(\theta)dr\wedge d\phi$$

$$H_{tr}^\Lambda = e^{2U-2\psi}I^{\Lambda\Sigma}(R_{\Lambda\Gamma}p^\Gamma - q_\Sigma),\quad H_{\theta\phi}^\Lambda = p^\Lambda f_l(\theta)\,.$$

- Now ***all the fields*** defining this ansatz ***depend only on the radial coordinate*** r***!***

Effective field theory

- The substitution of this ansatz in the e.o.m. shows the possibility to derive these equations from a finite dimensional ***dynamical effective system***

$$S_{eff} = \int dr[e^{2\psi}(U'^2 - \psi'^2 + h_{uv}q'^u q'^v + g_{i\bar{j}}z'^i \bar{z}'^{\bar{j}}$$

$$+\frac{1}{4}e^{4U-4\psi}\mathscr{Q}'\mathscr{H}^{-1}\mathscr{Q}') - V_{eff}],$$

with

$$V_{eff} = -\left(e^{2U-2\psi}V_{BH} + e^{2\psi-2U}V_g - l\right).$$

ensuring the constaints $H_{eff} = 0$ and $p^\Lambda k^u_\Lambda - q_\Lambda k^{u\Lambda} = 0$

- The e.o.m. now are switched ***from PDE to ODE...***
- ...however highly coupled and of the second order***...we can do something better!***

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Hamilton-Jacobi

- For a ***n-dimensional dynamical system*** like

$$S = \int dr \left(\frac{1}{2} g_{ab} \Phi'^a \Phi'^b - V(\Phi^a) \right) ,$$

with the condition $H = 0$, Hamilton-Jacobi equation for the principal function $W = W(\Phi^a; \alpha^1, ..., \alpha^n)$ reads

$$\frac{1}{2} g^{ab} \partial_a W \partial_b W = -V(\Phi^a) .$$

- In general, too difficult to find the complete integral...easier to have a particular solution!
- In this case we have not solved completly the system, however we can ***rewrite the action as sum of square***

$$S = \int dr \frac{1}{2} g_{ab} \left(\Phi'^a \pm g^{ac} \partial_c W \right) \left(\Phi'^b \pm g^{bd} \partial_d W \right) .$$

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First order equations

Applaing this technique to S_{eff} showed before, we find:

$$U' = \varepsilon(e^{U-2\psi}Re\tilde{\mathscr{Z}} + le^{-U}Im\tilde{\mathscr{L}}),$$
$$\psi' = 2l\varepsilon e^{-U}Im\tilde{\mathscr{L}},$$
$$z'^i = \varepsilon e^{i\alpha}g^{i\bar{j}}\left(e^{U-2\psi}\bar{D}_{\bar{j}}\bar{\mathscr{Z}} - ile^{-U}\bar{D}_{\bar{j}}\bar{\mathscr{L}}\right),$$
$$q'^u = -2l\varepsilon h^{uv}e^{-U}Im(e^{-i\alpha}\partial_v\mathscr{L}),$$
$$\mathscr{Q}' = \varepsilon 4e^{2\psi-3U}\mathscr{H}\Omega Re\tilde{\mathscr{V}},$$

$$2e^U\langle Re\tilde{\mathscr{V}},\mathscr{K}^u\rangle = -\varepsilon\langle\mathscr{A},\mathscr{K}^u\rangle \quad \langle\mathscr{K}^u,\mathscr{P}^x\rangle = 0,$$

Where the *principal Hamilton-Jacobi* function reads

$$W = e^U|\mathscr{Z} + ile^{2\psi-2U}\mathscr{Q}^x\mathscr{W}^x|,$$

- We have generalized the result of BPS analysis of Halmagyi, Petrini, Zaffaroni [$arXiv: 1305.0730v3[hep-th]$] and Dall'Agata, Gnecchi [$arXiv: 1012.37565v1[hep-th]$].

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2. We have seen a ***formal development of the Einstein equations*** aims to find out new solutions.

...are ***both part of our work***!

- Outlook
- The next step is to solve the equations in some ***particular models and further extend the result*** to a large class of black holes: non-extremal, rotating and NUT-charged...
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