IMPROVING THE UNDERSTANDING AND EFFICIENCY OF PDF COMPUTATIONS

First year PhD Workshop

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OUTLINE

Parton Distribution Functions from Quantum Chromodynamycs

Neural Networks PDFs

Representation of PDF uncertainties

PARTON DISTRIBUTION FUNCTIONS FROM QUANTUM CHROMODYNAMYCS

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- But equations cannot be solved exactly.

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- In the low energy limit QCD exhibits *non-perturbative* effects.
    - Quark confinment

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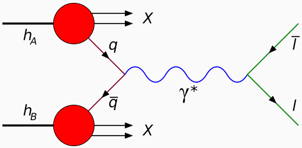

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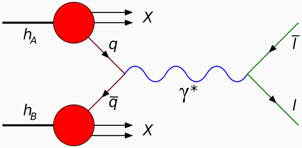
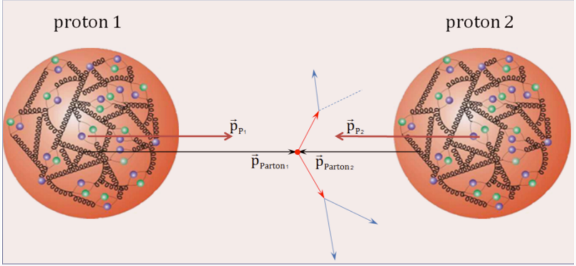


- Parton Distribution Functions (PDFs) relate the high energy parton interaction to the hadron structure.

To first order in α_S PDFs are probability densities

Probability of sampling a given parton (quark u, quark d, gluon...) with a given momentum $\vec{p}_{parton}$.



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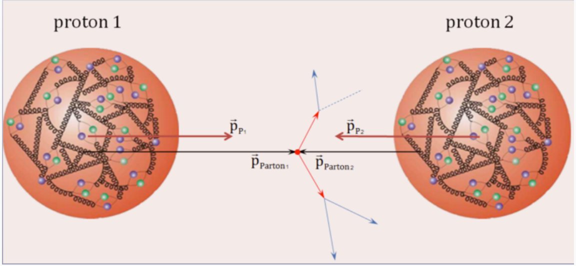

- At higher orders affected by quantum corrections (see next talk by Claudio Muselli).

NEURAL NETWORKS PDFS

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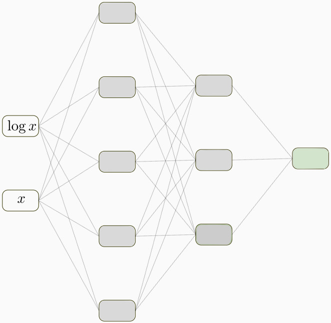
- We don’t have strong reasons to assume any particular functional form for the PDFs.

- We provide a PDF determination based on Neural Networks.
- We strive to obtain a statistically consistent and unbiased result incorporating all relevant experimental data.

NEURAL NETWORK

- General class of function constructed from iterarive composition of simple functions.
  - At each node we apply a function to a linear combination of inputs.
  - The coefficients of each combination are the *parameters* ($37 \times 7\,\text{PDFs} = 259$ for NNPDF).



$$-\theta_{30} + \frac{w^{30}_{20}}{e^{\theta_{20} - \frac{w^{20}_{10}}{e^{\theta_{10} - w^{10}_{00} x_{00} - w^{10}_{01} \log(x_{00})} + 1} - \frac{w^{20}_{11}}{e^{\theta_{11} - w^{11}_{00} x_{00} - w^{11}_{01} \log(x_{00})} + 1} - \frac{w^{20}_{12}}{e^{\theta_{12} - w^{12}_{00} x_{00} - w^{12}_{01} \log(x_{00})} + 1} - \frac{w^{20}_{13}}{e^{\theta_{13} - w^{13}_{00} x_{00} - w^{13}_{01} \log(x_{00})} + 1} - \frac{w^{20}_{14}}{e^{\theta_{14} - w^{14}_{00} x_{00} - w^{14}_{01} \log(x_{00})} + 1}} + 1}$$

$$+ \frac{w^{30}_{21}}{e^{\theta_{21} - \frac{w^{21}_{10}}{e^{\theta_{10} - w^{10}_{00} x_{00} - w^{10}_{01} \log(x_{00})} + 1} - \frac{w^{21}_{11}}{e^{\theta_{11} - w^{11}_{00} x_{00} - w^{11}_{01} \log(x_{00})} + 1} - \frac{w^{21}_{12}}{e^{\theta_{12} - w^{12}_{00} x_{00} - w^{12}_{01} \log(x_{00})} + 1} - \frac{w^{21}_{13}}{e^{\theta_{13} - w^{13}_{00} x_{00} - w^{13}_{01} \log(x_{00})} + 1} - \frac{w^{21}_{14}}{e^{\theta_{14} - w^{14}_{00} x_{00} - w^{14}_{01} \log(x_{00})} + 1}} + 1}$$

$$+ \frac{w^{30}_{22}}{e^{\theta_{22} - \frac{w^{22}_{10}}{e^{\theta_{10} - w^{10}_{00} x_{00} - w^{10}_{01} \log(x_{00})} + 1} - \frac{w^{22}_{11}}{e^{\theta_{11} - w^{11}_{00} x_{00} - w^{11}_{01} \log(x_{00})} + 1} - \frac{w^{22}_{12}}{e^{\theta_{12} - w^{12}_{00} x_{00} - w^{12}_{01} \log(x_{00})} + 1} - \frac{w^{22}_{13}}{e^{\theta_{13} - w^{13}_{00} x_{00} - w^{13}_{01} \log(x_{00})} + 1} - \frac{w^{22}_{14}}{e^{\theta_{14} - w^{14}_{00} x_{00} - w^{14}_{01} \log(x_{00})} + 1}} + 1}$$

FITTING PROCEDURE

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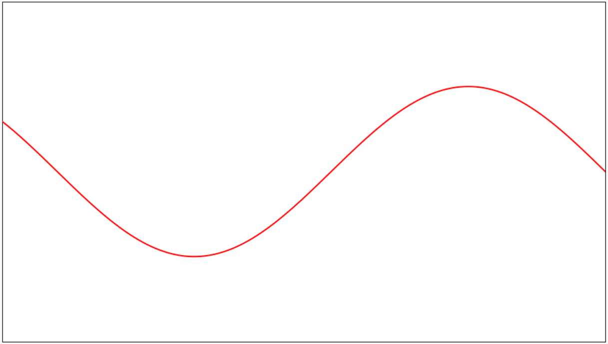
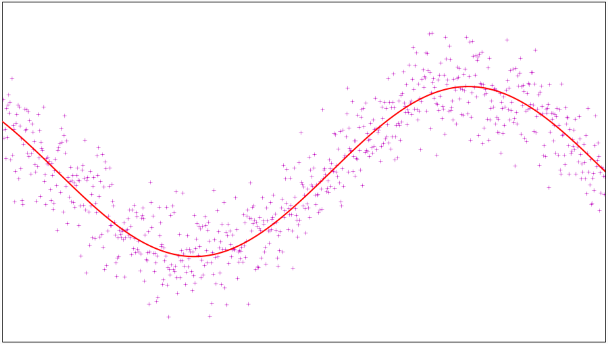
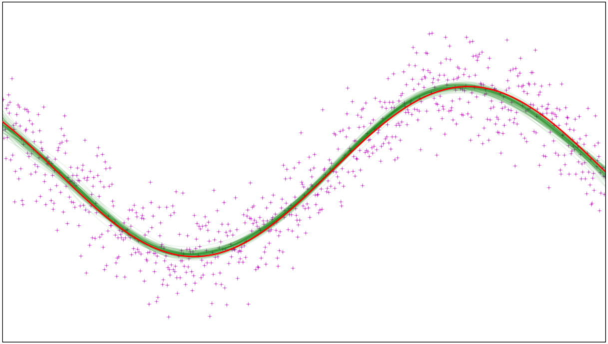
- Use a genetic algorithm to fit parameters.
- Cross validation is used to avoid *overlearning* (i.e fitting experimental noise).
- We repeat the procedure many times to obtain a *Monte Carlo Ensemble* of functions representing uncertainty.



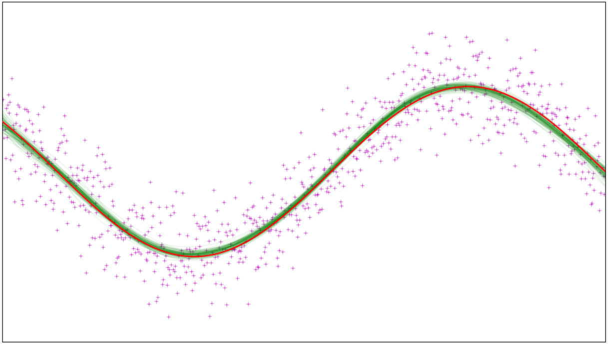
Assume we know the underlying function



Generate fake data



Apply our fitting procedure



Verify statistical consistency

REPRESENTATION OF PDF UNCERTAINTIES

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- A Hessian (Multigaussian) representation is advantageous (trough less general and precise)

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- We derived a method, `mc2hessian`, for converting from Monte Carlo to Hessian representation (arxiv:1505.06736, doi:10.1140/epjc/s10052-015-3590-7) .

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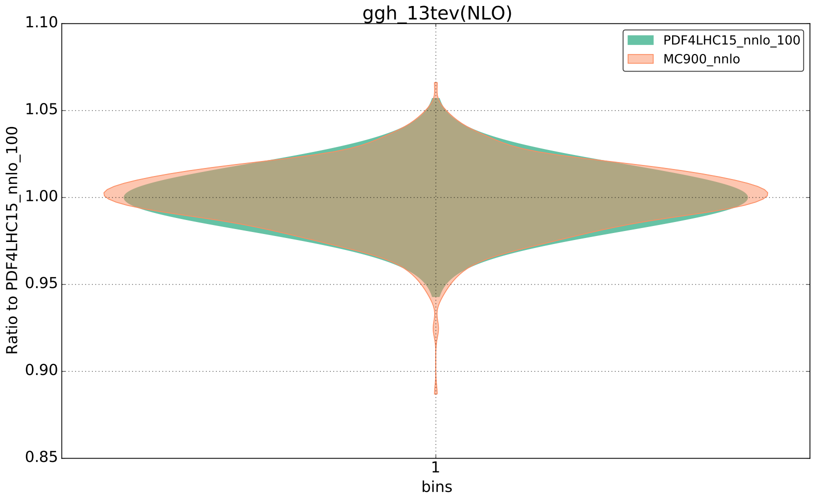
- Significative reduction in the number of computations needed (about a factor 10).
- Derive a more advanced iterative procedure to further improve computations for specific processes, giving another factor 10 (following soon).

EXAMPLE


ggh_13tev(NLO)
PDF4LHC15_nnlo_100
MC900_nnlo
Ratio to PDF4LHC15_nnlo_100
1.10
1.05
1.00
0.95
0.90
0.85
1
bins

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- `mc2hessian` is used as a combination method.

Thank you!