

Let us put $J = 1$.

The eigenvalues of the transfer matrix are:

$$\begin{aligned}\tau_+(\beta, B) &= e^\beta \left(\sqrt{e^{-4\beta} + \sinh^2(\beta B)} + \cosh(\beta B) \right) \\ \tau_-(\beta, B) &= e^\beta \left(\cosh(\beta B) - \sqrt{e^{-4\beta} + \sinh^2(\beta B)} \right)\end{aligned}\tag{1}$$

The partition function reads:

$$Z(N, \beta, B) = \tau_+(\beta, B)^N + \tau_-(\beta, B)^N\tag{2}$$

and the magnetization $M(N, \beta, B) = \frac{1}{\beta} \partial_B \log(Z(N, \beta, B))$ is:

$$\frac{e^{-\beta N} \sinh(\beta B) \left(\left(e^\beta \left(\cosh(\beta B) - \sqrt{e^{-4\beta} + \sinh^2(\beta B)} \right) \right)^N - \left(e^\beta \left(\sqrt{e^{-4\beta} + \sinh^2(\beta B)} + \cosh(\beta B) \right) \right)^N \right)}{\sqrt{e^{-4\beta} + \sinh^2(\beta B)} \left(\left(\cosh(\beta B) - \sqrt{e^{-4\beta} + \sinh^2(\beta B)} \right)^N + \left(\sqrt{e^{-4\beta} + \sinh^2(\beta B)} + \cosh(\beta B) \right)^N \right)}\tag{3}$$

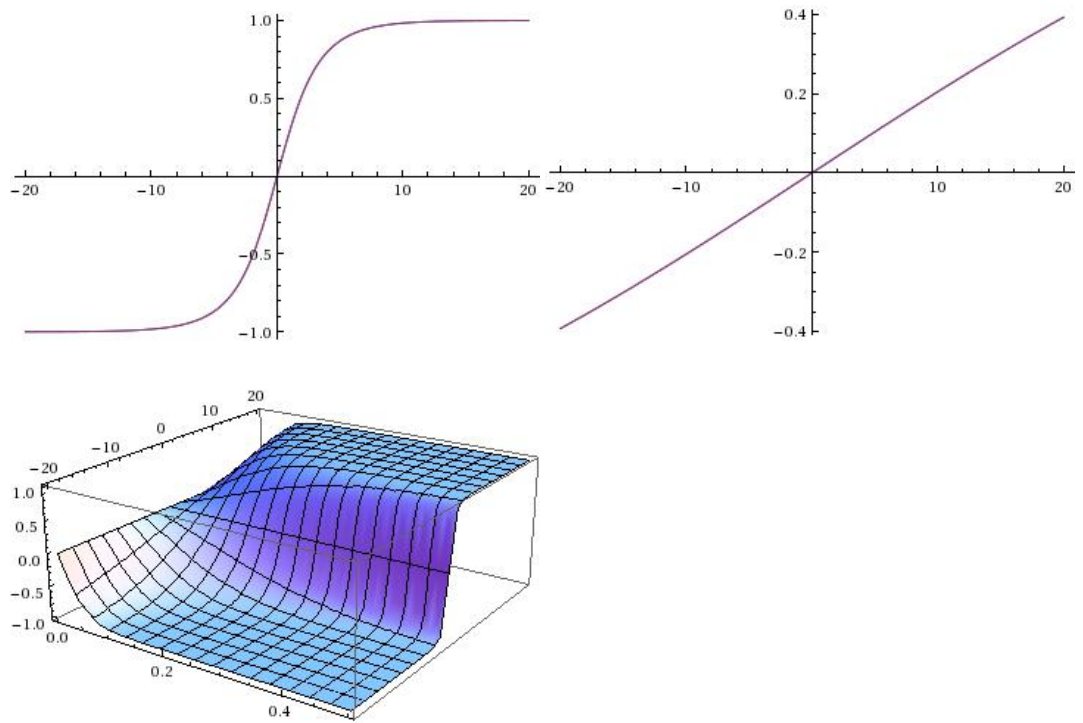


Figure 1: (1): magnetization at $\beta = 0.2$, $N = 10, \infty$ as a function of $B \in (-20, 20)$. (2): magnetization at $\beta = 0.02$, $N = 10, \infty$ as a function of $B \in (-20, 20)$. (3): magnetization as a function of β, B at $N = \infty$