Phd School in Physics, Astrophysics and Applied Physics

End Year Seminar

About the use of fidelity the access quantum resources

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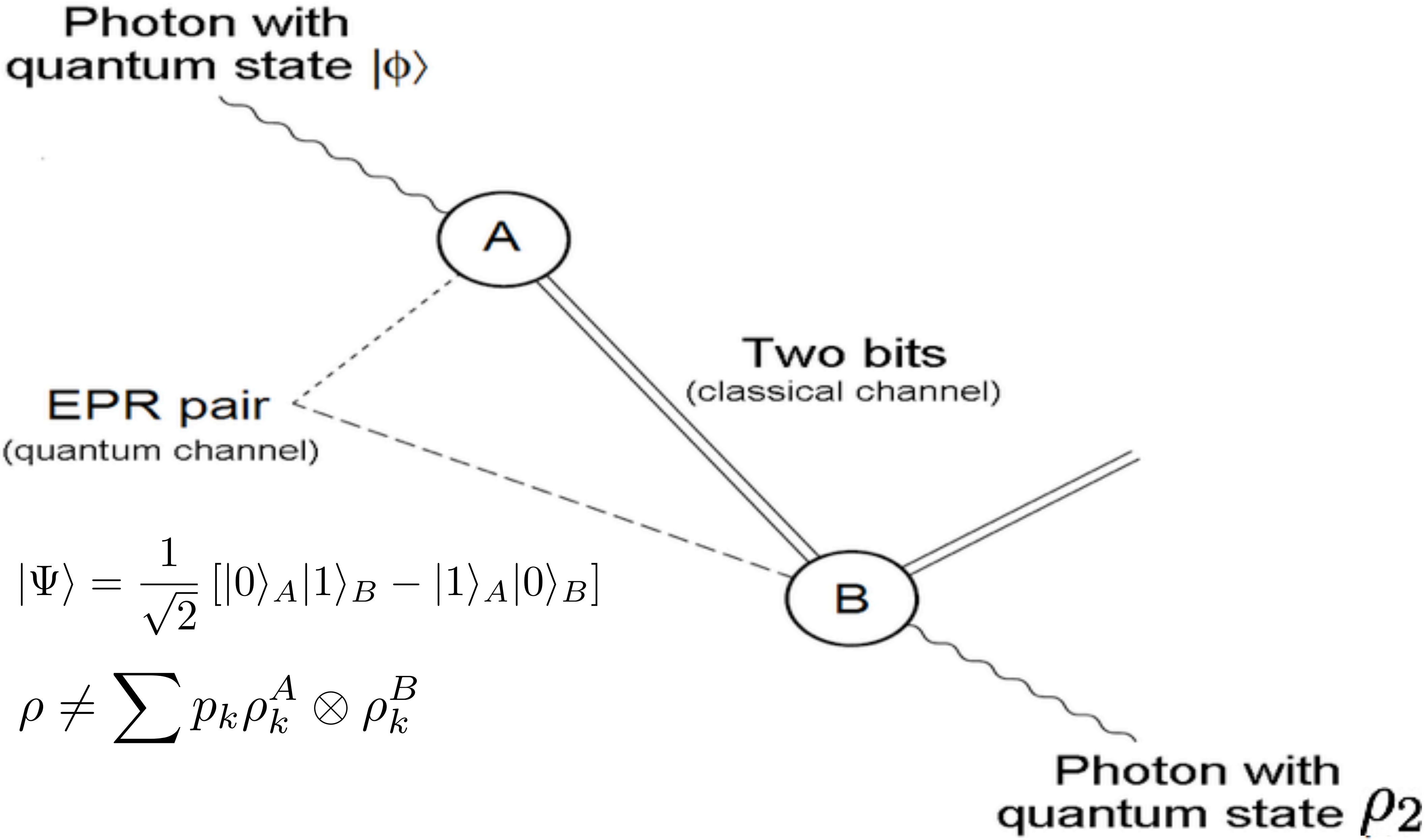
Quantum Teleportation



$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left[|0\rangle_A|1\rangle_B - |1\rangle_A|0\rangle_B\right]$$

$$\rho \neq \sum p_k \rho_k^A \otimes \rho_k^B$$

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In cauda venenum

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We found examples where this is not quite true if "fidelity close to unit" is intended to mean the usual values 0.9, 0.99 or so.

Single-mode Gaussian State

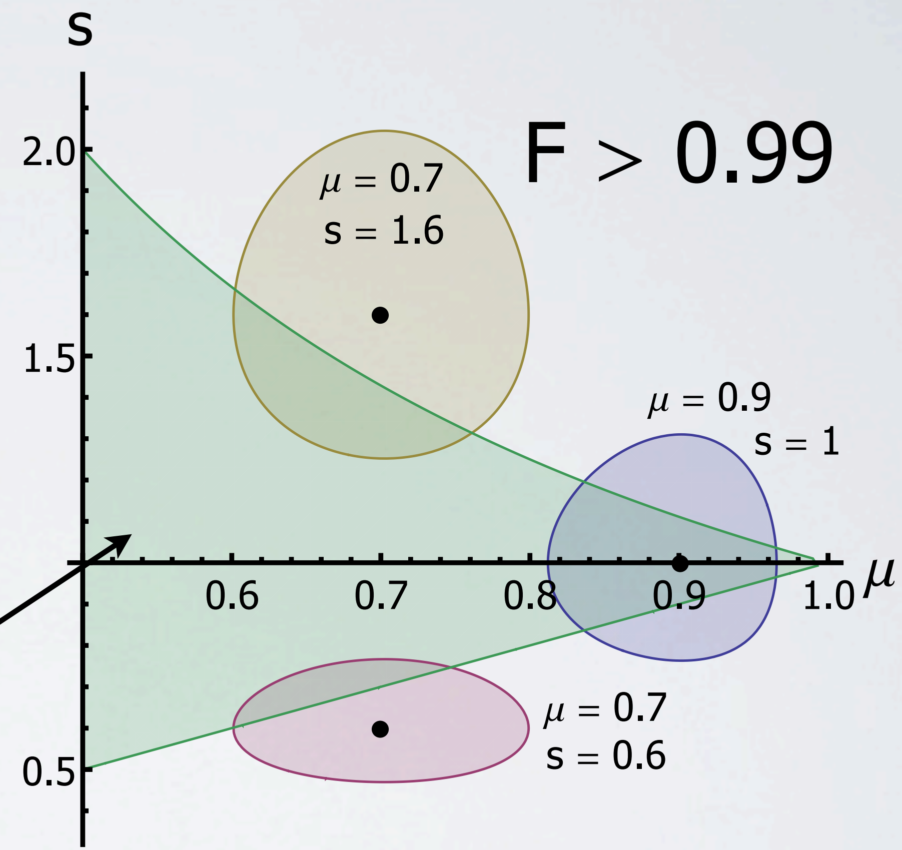
$$\rho_{s\mu} = S(r)\nu(N)S^{\dagger}(r)$$

$$\nu(N) = \frac{N^{a^{\dagger}a}}{(1+N)^{a^{\dagger}a}}$$

$$S(r) = \exp\{r(a^{\dagger 2} - a^2)\}$$

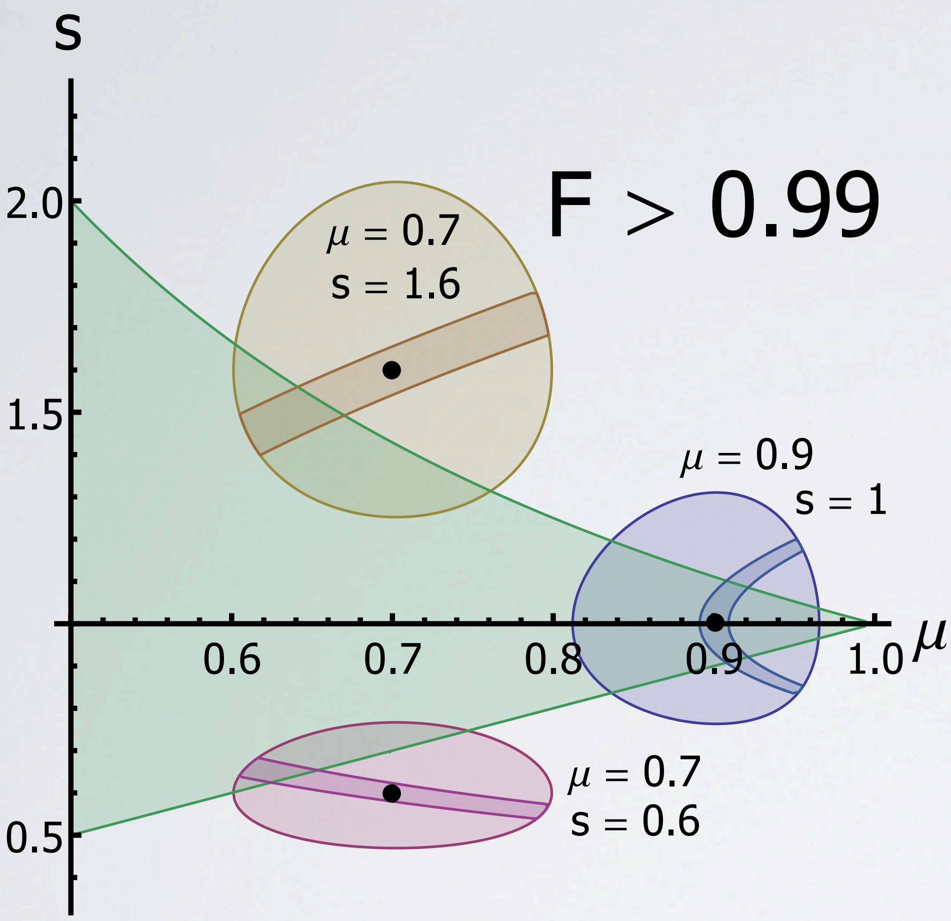
$$s = e^{2r}$$

$$\mu = \frac{1}{2N+1}$$

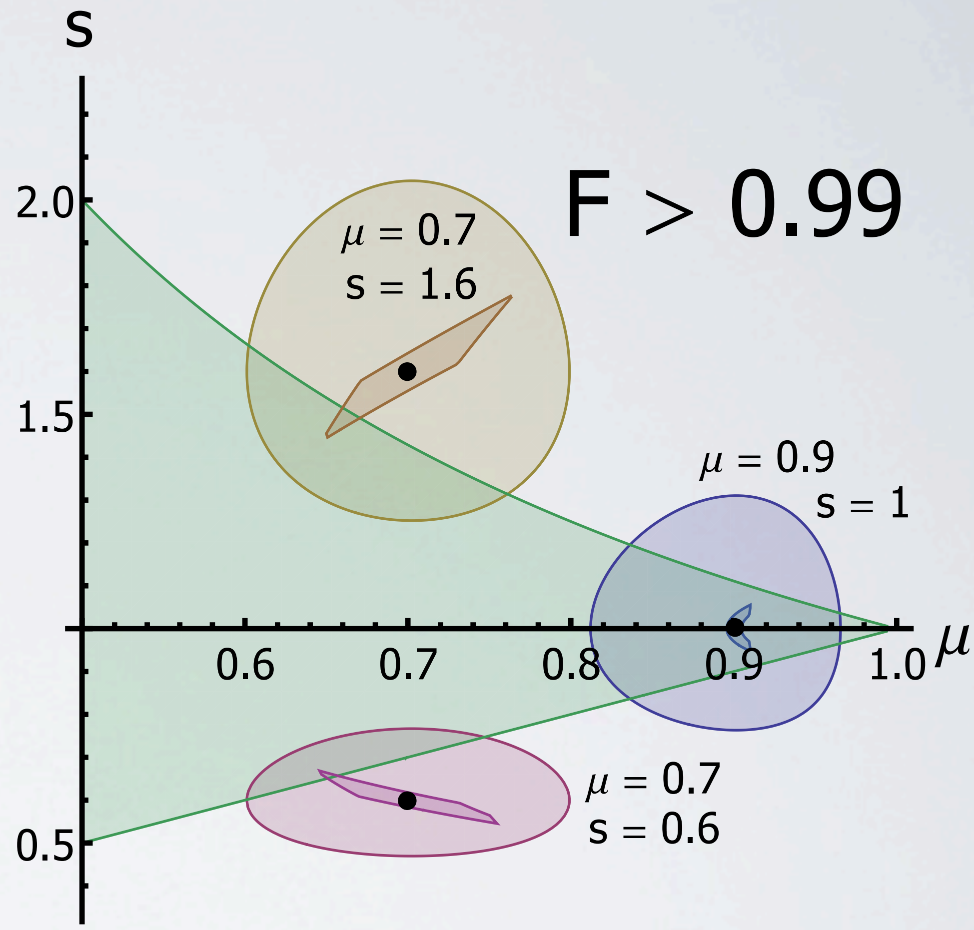


Region of classicality region
Singular Glauber P-function

Is a cure achievable?



mean photon number differing at most 10%

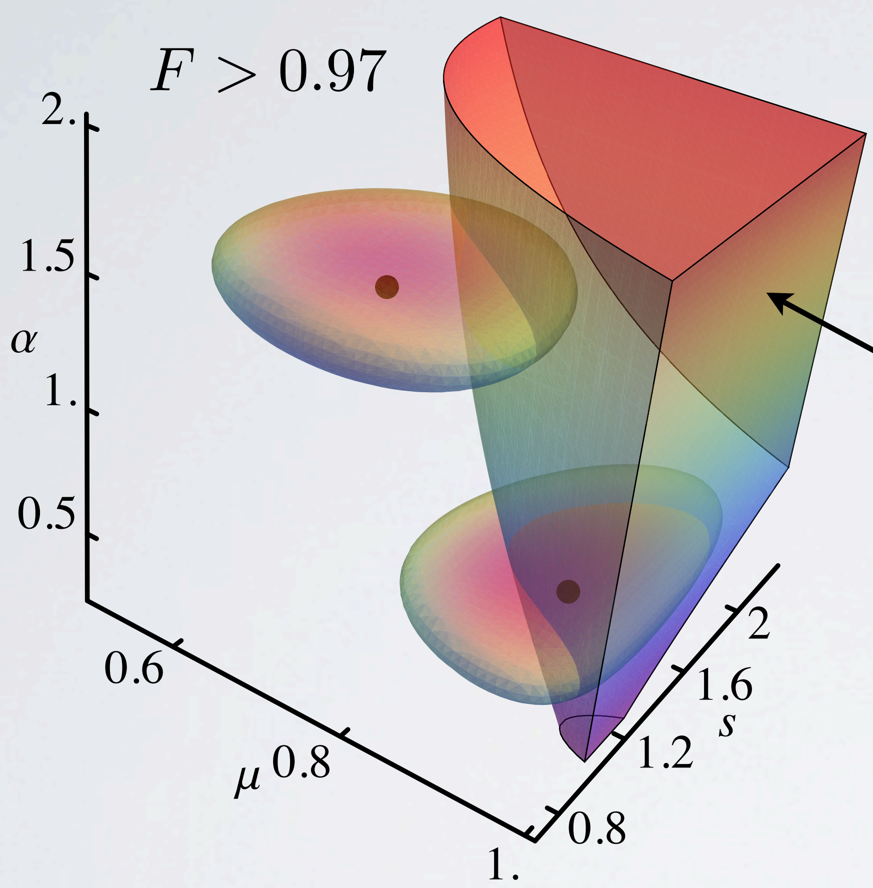


mean photon number differing at most 10%

&

photon number fluctuations differing at most 10%

Variation on a theme



$$\rho_G = D(x)\rho_{s\mu}D^\dagger(x)$$

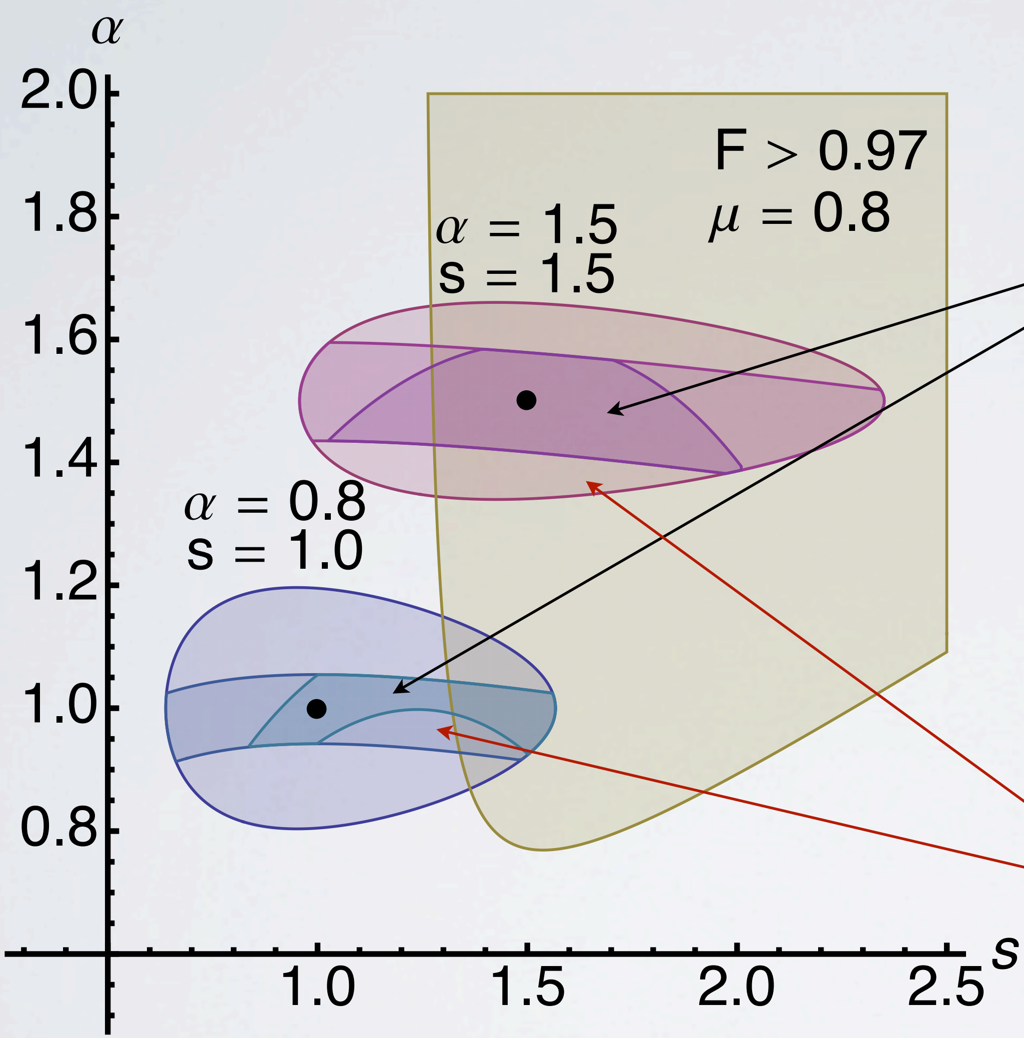
$$D(x) = \exp\{\alpha a^\dagger - \bar{\alpha} a\}$$

Fano Factor

$$R = \frac{\langle \Delta n^2 \rangle}{n} < 1$$

superPoissonian Target State $\mu = 0.7 \quad s = 1.2 \quad \alpha = 1.5$

subPoissonian Target State $\mu = 0.9 \quad s = 1.4 \quad \alpha = 0.5$



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Two-mode Gaussian State

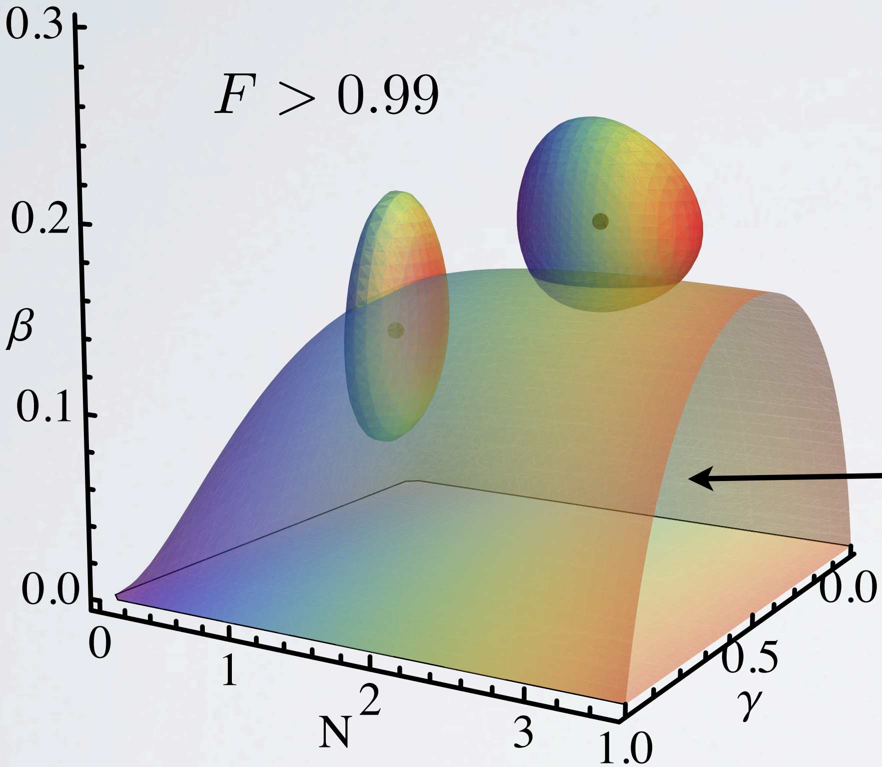
$$\rho_{N\beta\gamma} = S_2(r)\nu_1(N_1) \otimes \nu_2(N_2) S_2^\dagger(r)$$

$$S_2(r) = \exp\{r(a^\dagger b^\dagger - ab)\}$$

$$N = N_1 + N_2 + 2(1 + N_1 + N_2)N_s$$

$$\beta = N_s/N$$

$$\gamma = \frac{N_1}{N_1 + N_2}$$



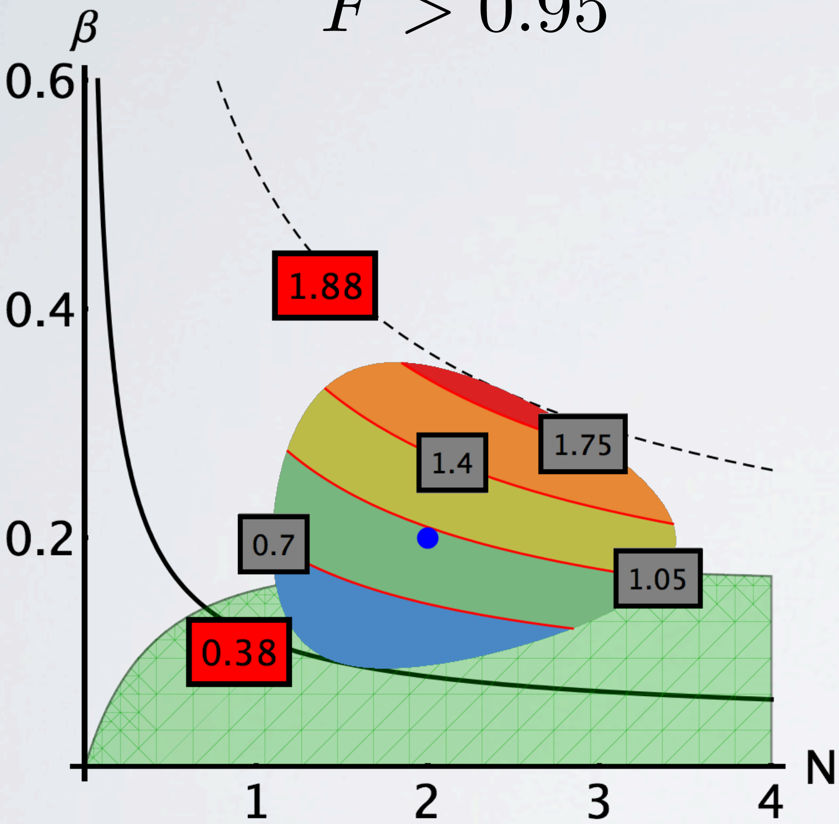
Separability Region (PPT Criterion)

Entangled Target State $N = 2.5 \quad \beta = 0.2 \quad \gamma = 0.5$

Separable Target State $N = 1 \quad \beta = 0.13 \quad \gamma = 0.5$

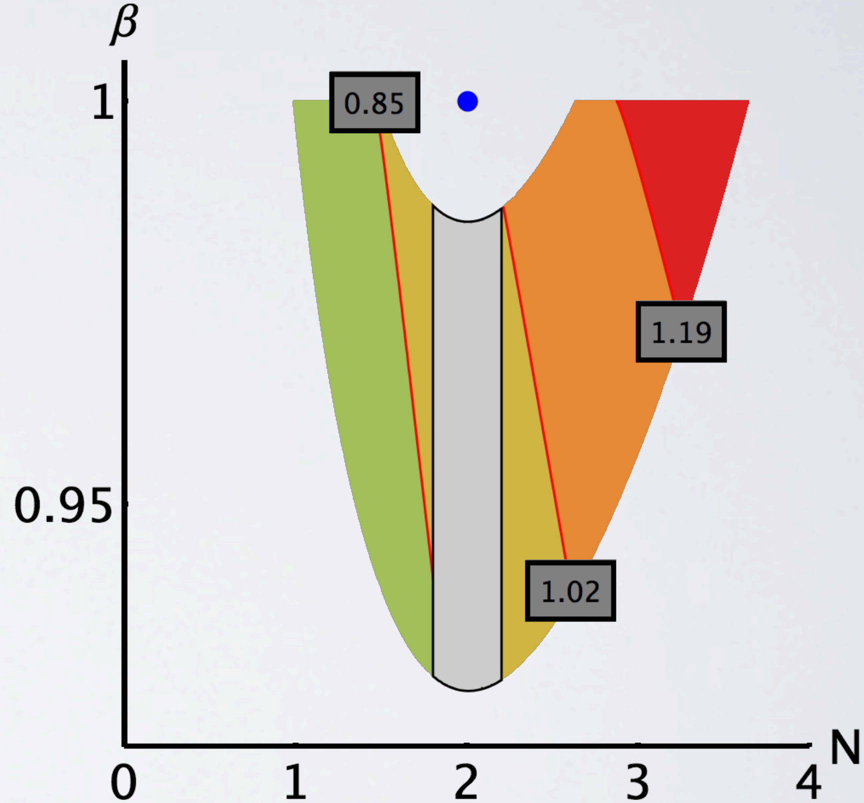
Fidelity And Discord

$F > 0.95$



$N = 2 \quad \beta = 0.2 \quad \gamma = 0.5$

$0.95 < F < 0.99$



$N = 2 \quad \beta = 1$

Conclusion And Outlooks

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Which is the minimal set of quantities to be specified in order to certify quantum resources?

If you would like to know more of it, please ask questions in the next 5 mins or feel free to come upstair (5th floor) or take a look at arXiv:quant-ph/1309.5325

Thanks for your attention