

# Playing Billiards with Microwaves

## Quantum Manifestations of Classical Chaos



TECHNISCHE  
UNIVERSITÄT  
DARMSTADT

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2011**

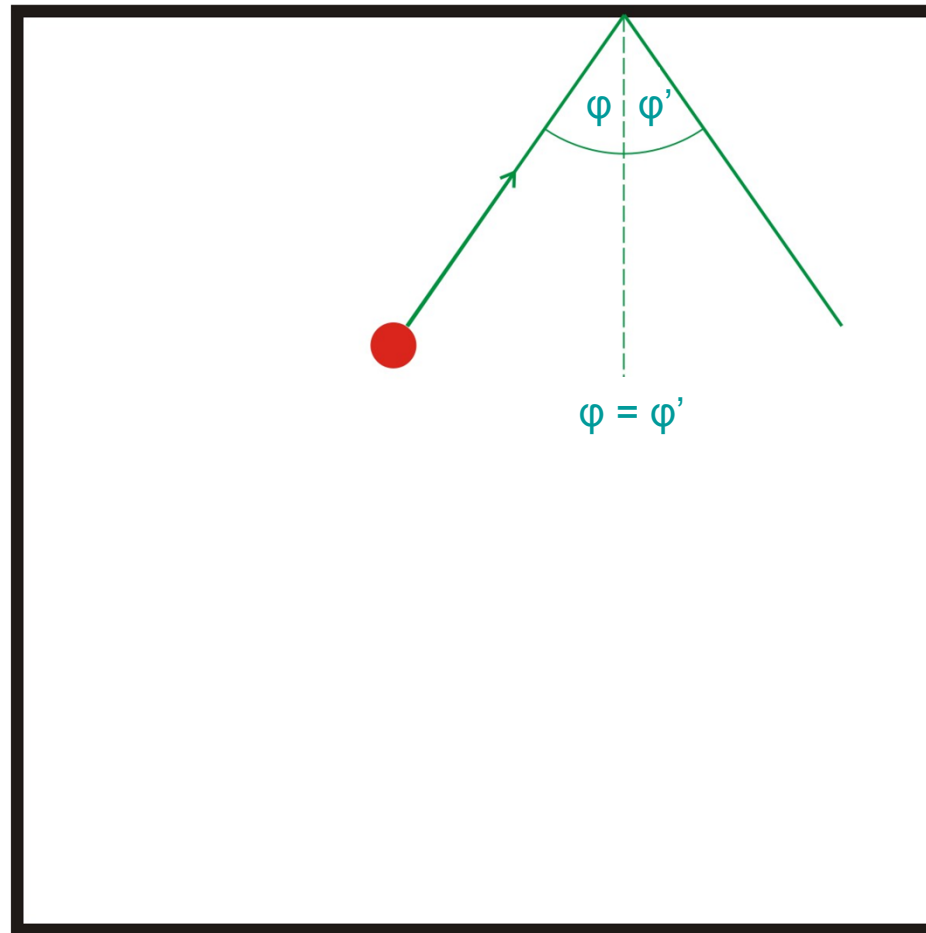
- Classical billiards and quantum billiards
- Random Matrix Theory (Wigner 1951 – Dyson 1962)
- Spectral properties of billiards and mesoscopic systems
- Two examples:
  - Spectra and wavefunctions of mushroom billiards
  - Friedel oscillations in microwave cavities
- Chaotic scattering: S-matrix fluctuations in the regime of overlapping resonances for T invariant and T non-invariant systems

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H.L. Harney, S. Tomsovic, H.A. Weidenmüller

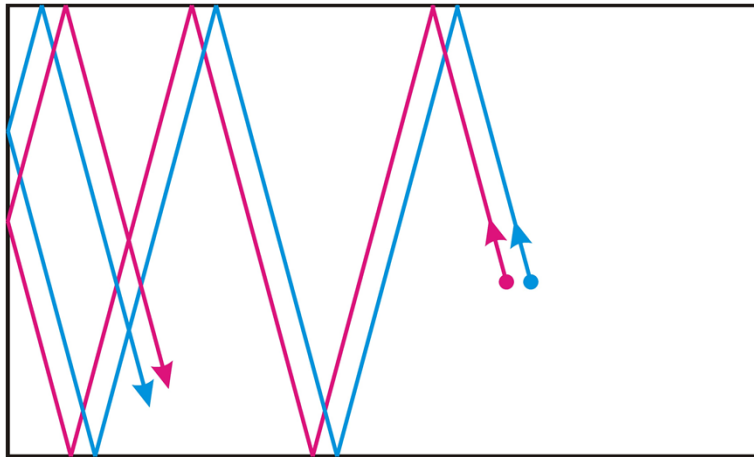


# Classical Billiard



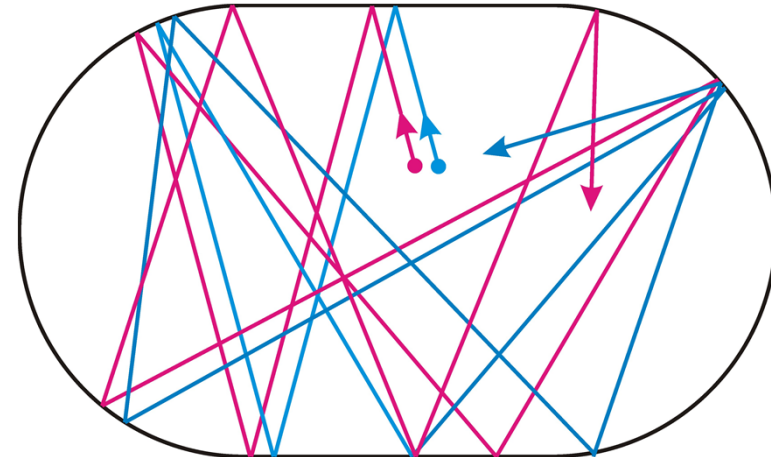
# Regular and Chaotic Dynamics

Regular



- Energy and  $p_x^2$  are conserved
- Equations of motion are integrable
- Predictable for infinite long times

Bunimovich stadium (chaotic)

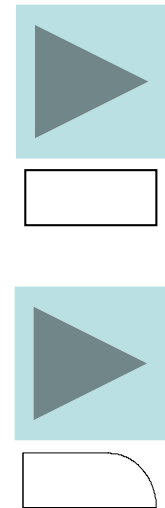
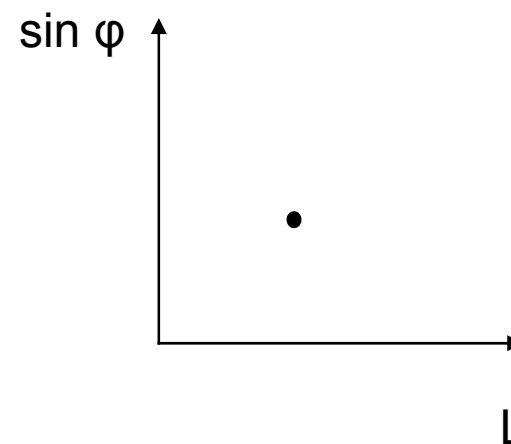
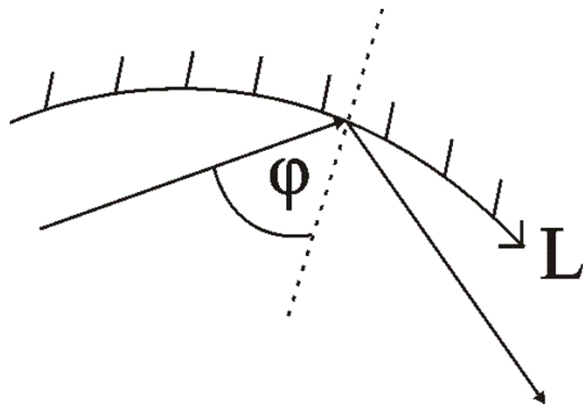


- Only energy is conserved
- Equations of motion are not integrable
- Predictable for a finite time only

# Tool: Poincaré Sections of Phase Space

- Parametrization of billiard boundary:  $L$
- Momentum component along the boundary:  $\sin \varphi$

} conjugate variables



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# Small Changes → Large Actions

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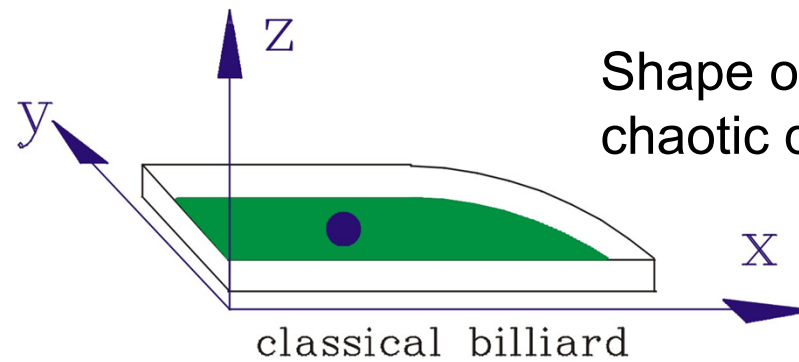
- Sensitivity of the solutions of a deterministic problem with respect to small changes in the initial conditions is called **Deterministic Chaos**.
- Beyond a fixed, for the system **characteristic time** every prediction becomes impossible. The system behaves in such a way as if not determined by physical laws but randomness.



# Our Main Interest

- How are these properties of classical systems transformed into corresponding quantum-mechanical systems?
- Note: In QM distinction between integrable and chaotic systems does not work any longer:
  - Because of  $\Delta x \Delta p \geq \hbar / 2$  the concept of trajectories loses significance
  - Schrödinger equation is linear  $\rightarrow$  no chaos possible
- But: correspondence principle demands a relation between classical chaotic mechanics and quantum mechanics
  - $\rightarrow$  Quantum Chaos?
- What might we learn from generic features of billiards and mesoscopic systems (hadrons, nuclei, atoms, molecules, metal clusters, quantum dots) ?

# The Quantum Billiard and its Simulation



Shape of the billiard implies  
chaotic dynamics

# Schrödinger vs. Helmholtz Equation



quantum billiard

2D microwave cavity:  $h_z < \lambda_{\min}/2$

$$(\Delta + k^2)\Psi = 0$$

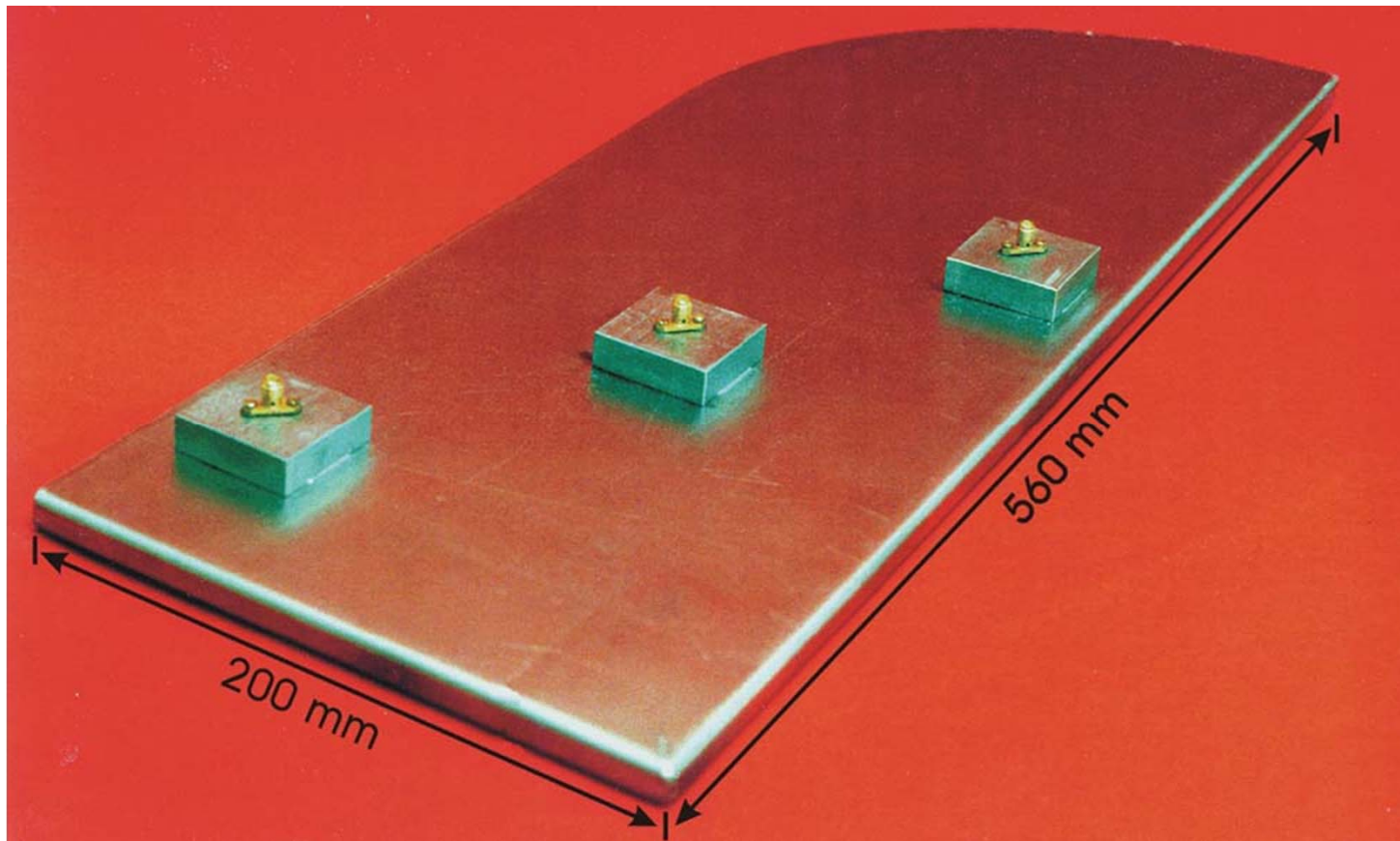
$$(\Delta + k^2)E_z = 0$$

Helmholtz equation and Schrödinger equation are equivalent in 2D. The motion of the quantum particle in its potential can be simulated by electromagnetic waves inside a two-dimensional microwave resonator.

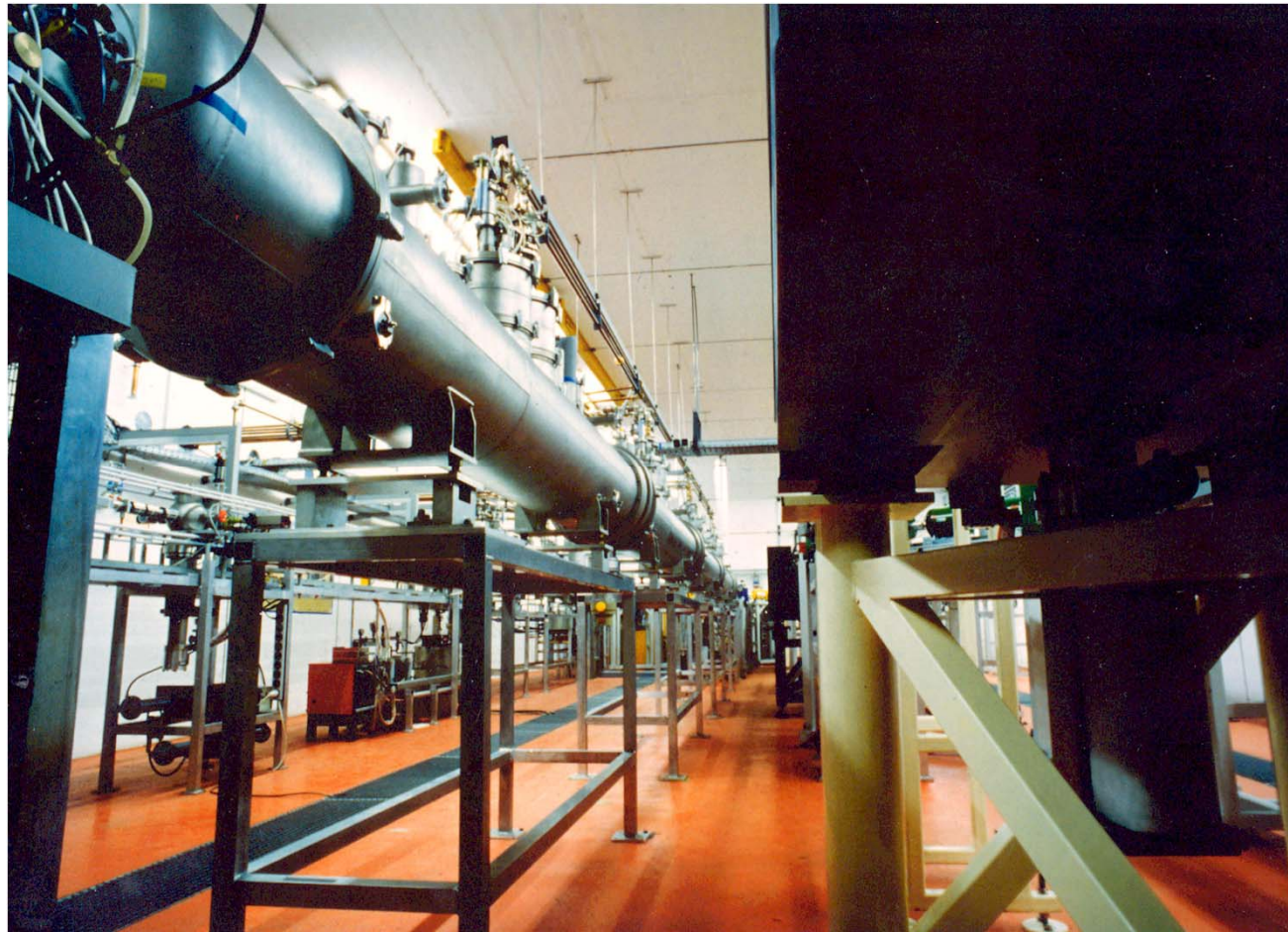




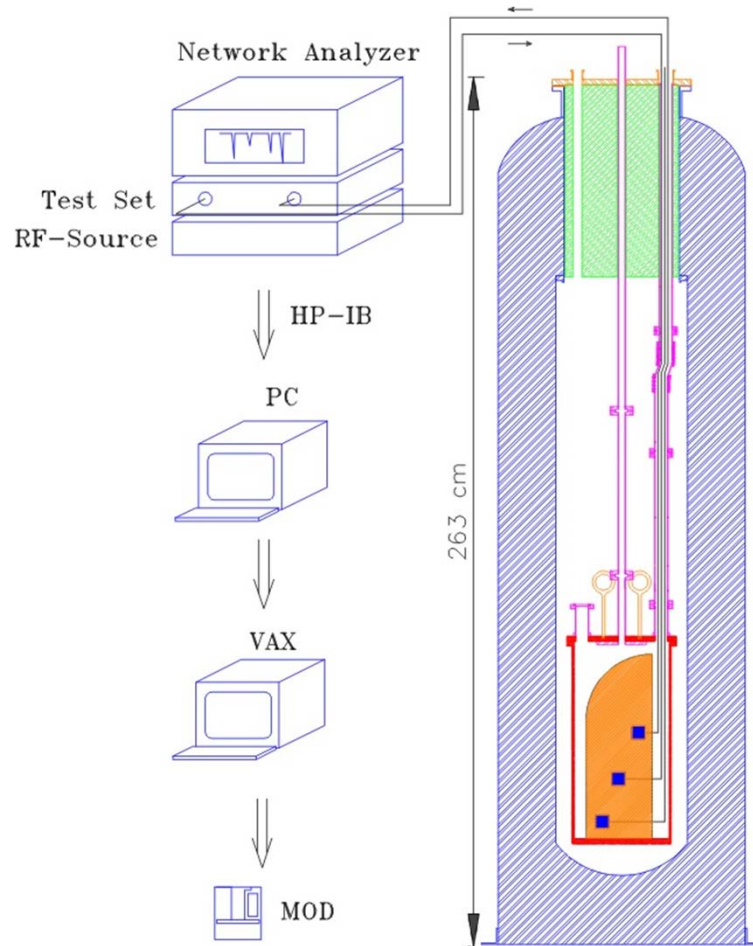
# Superconducting Niobium Microwave Resonator



# S-DALINAC



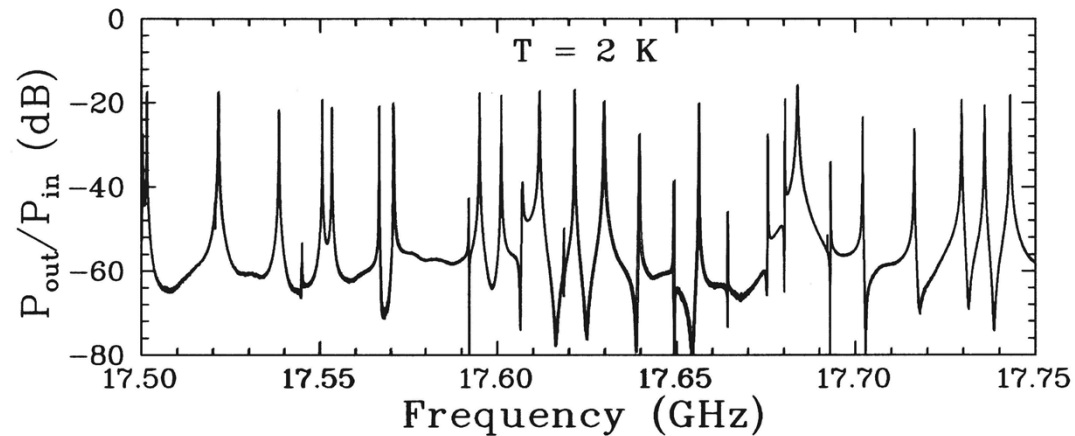
# Experimental Setup



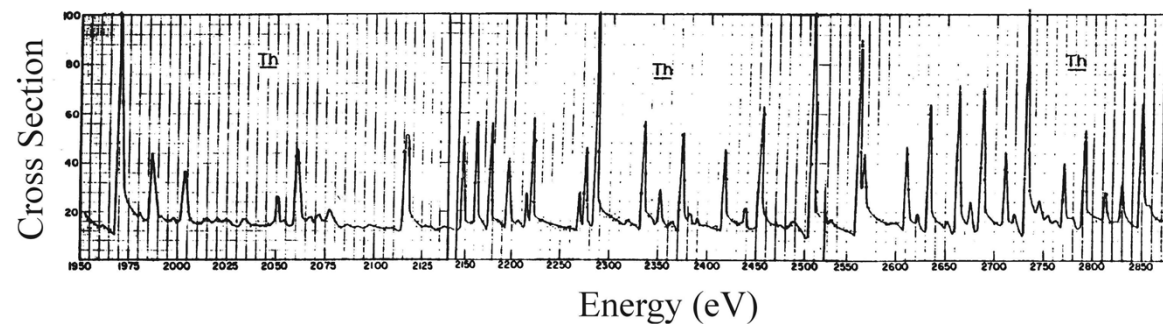
- Superconducting cavities
- LHe ( $T = 4.2 \text{ K}$ )
- $f = 45 \text{ MHz} \dots 50 \text{ GHz}$
- $10^3 \dots 10^4$  eigenfrequencies
- $Q = f/\Delta f \approx 10^6$

# Stadium Billiard $\leftrightarrow$ n + $^{232}\text{Th}$

Transmission spectrum for the stadium billiard



Spectrum of neutron resonances in  $^{232}\text{Th} + \text{n}$



# Niels Bohr's Model of the Compound Nucleus

The first of these is intended to convey an idea of events arising out of a collision between a neutron and the nucleus. Imagine a shallow basin with a number of billiard balls in it as shown in the accompanying figure. If the basin were empty, then upon striking a ball from the outside, it would go down one slope and pass out on the opposite side with its original velocity. But with other balls in the basin, there would not be a free passage of this kind. The struck ball would divide its energy first with one of the balls in the basin, these two would similarly share their energies with others, and so on until the original kinetic energy was divided among all the balls. If the basin and the balls are regarded as perfectly smooth and elastic, the collisions would continue until the kinetic energy happens again to be concentrated upon a ball close to the edge. This ball would then escape from the basin and the remainder of the balls would be left with insufficient total energy for any of them to climb the slope.

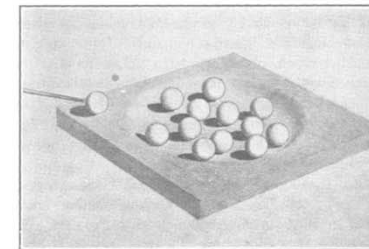


Fig. 1.

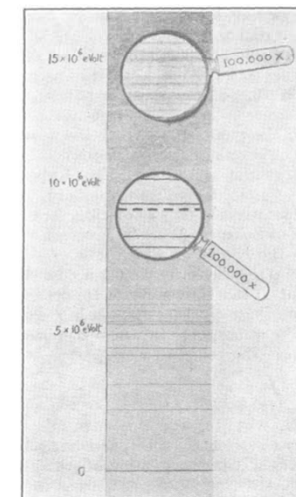


Fig. 2.

# Random Matrices $\leftrightarrow$ Level Schemes

Random  
Matrix

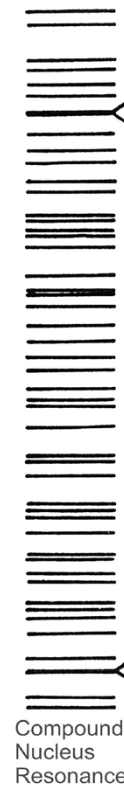
$$H = \begin{pmatrix} H_{11} & \dots & H_{1N} \\ \vdots & \ddots & \vdots \\ H_{N1} & \dots & H_{NN} \end{pmatrix}$$



Eigenvalues

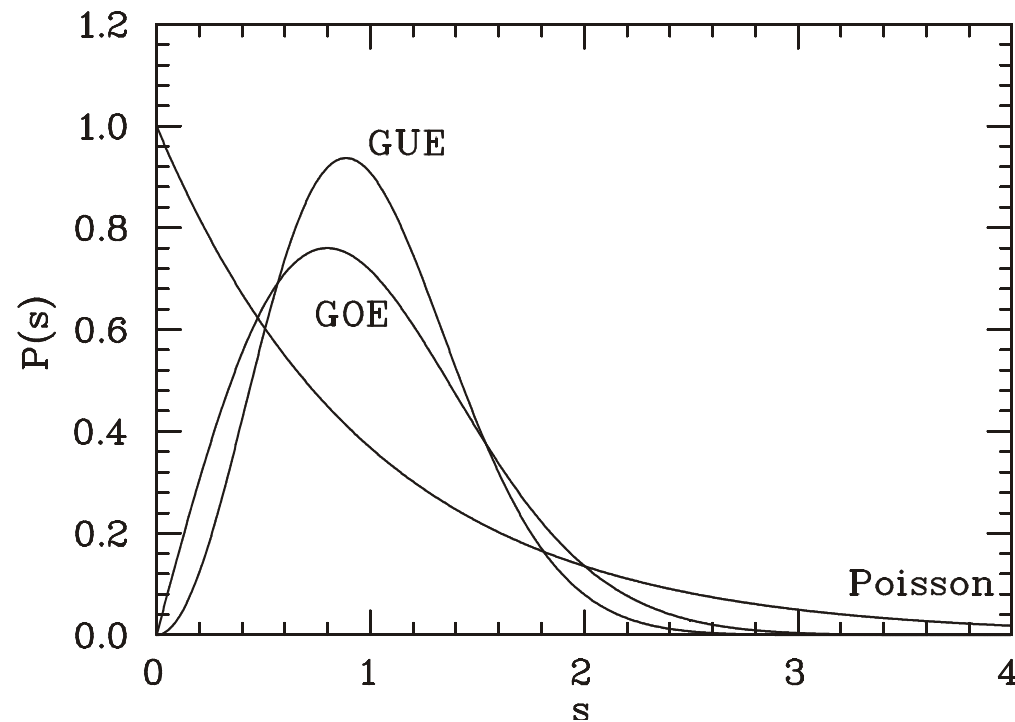
$$H\Psi_n = E_n\Psi_n \quad \longleftrightarrow$$

Level Schemes



GOE

# Nearest Neighbor Spacing Distribution



GOE and GUE



"Level Repulsion"

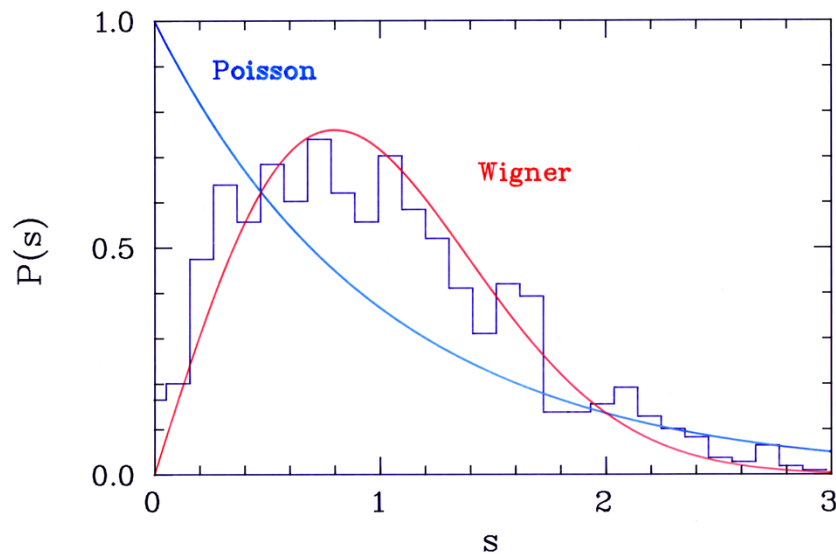
Poissonian Random Numbers



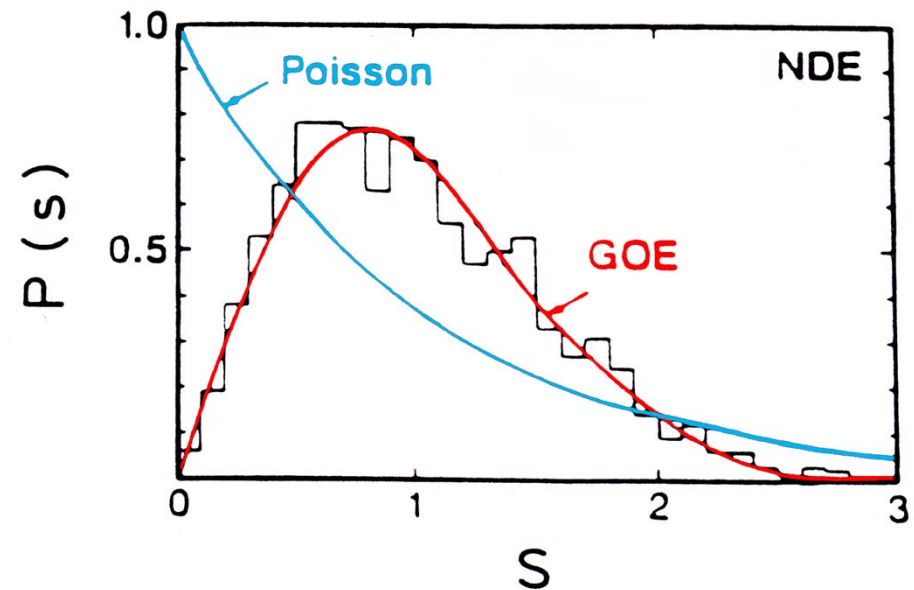
"Level Clustering"

# Nearest Neighbor Spacing Distribution

stadium billiard



nuclear data ensemble

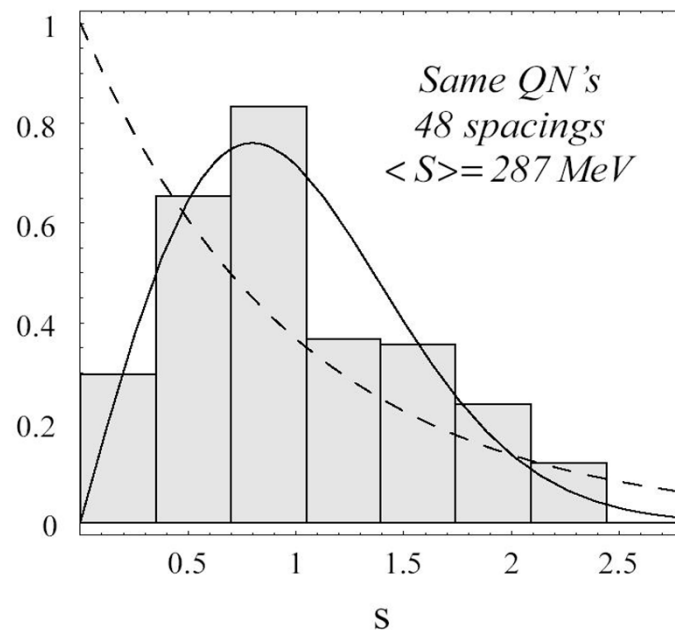


- Universal (generic) behaviour of the two systems



# Universality in Mesoscopic Systems: Quantum Chaos in Hadrons

- Combined data from measured baryon and meson mass spectra up to 2.5 GeV (from PDG)
- Spectra can be organized into multiplets characterized by a set of definite quantum numbers: isospin, spin, parity, strangeness, baryon number, ...

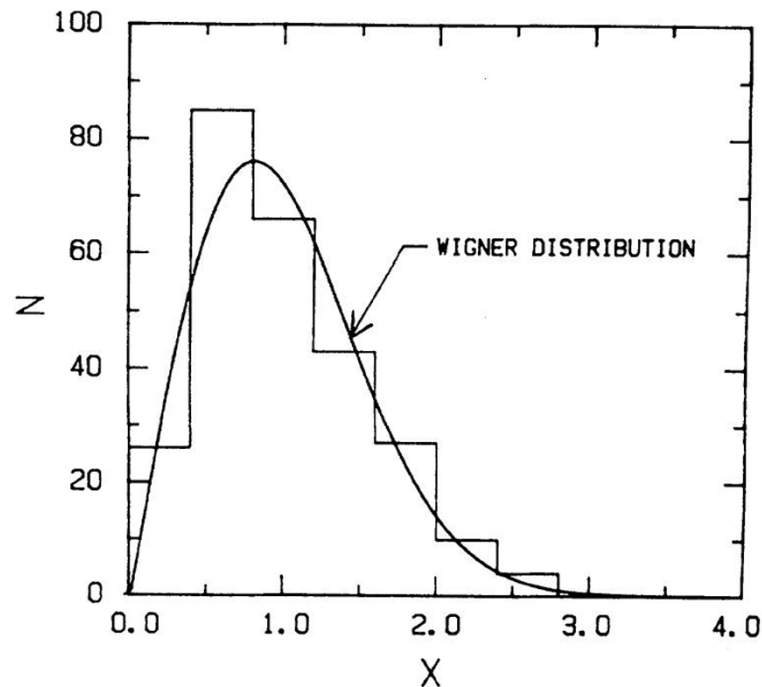


- Scale:  $10^{-16} \text{ m}$

Pascalutsa (2003)

# Universality in Mesoscopic Systems: Quantum Chaos in Atoms

- 8 sets of atomic spectra of highly excited neutral and ionized rare earth atoms combined into a data ensemble
- States of same total angular momentum and parity

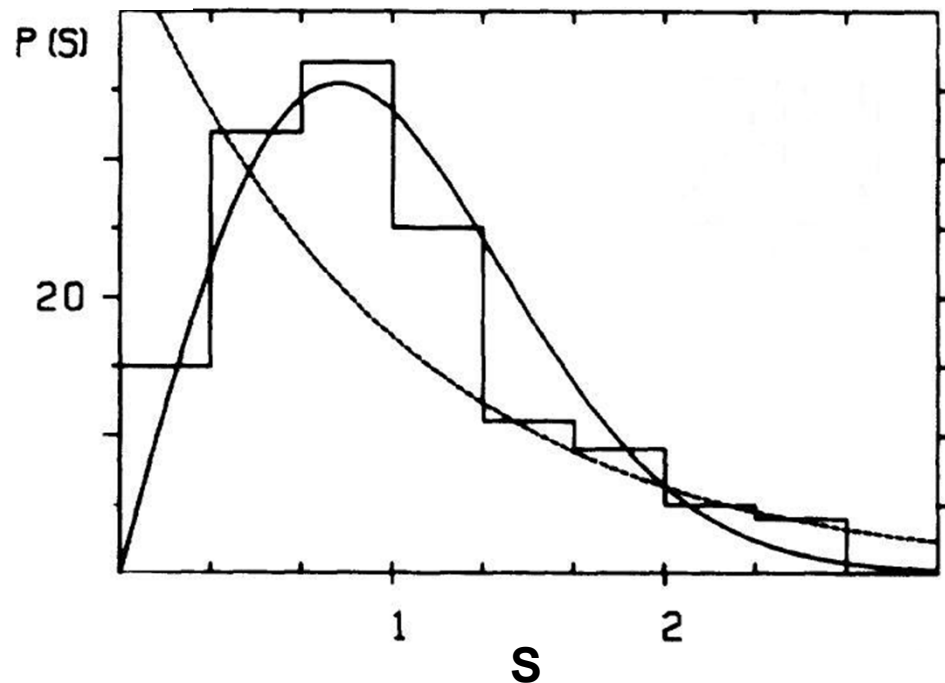


- Scale:  $10^{-10}$  m

Camarda + Georgopoulos (1983)

# Universality in Mesoscopic Systems: Quantum Chaos in Molecules

- Vibronic levels of  $\text{NO}_2$
- States of same quantum numbers



- Scale:  $10^{-9}$  m

Zimmermann et al. (1988)

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# How is the Behavior of the Classical System Transferred to the Quantum System?

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- There is a **one-to-one** correspondence between billiards and mesoscopic systems.

→ **Bohigas' conjecture:**

”The spectral properties of a generic **chaotic** system coincide with those of random matrices from the **GOE**“.

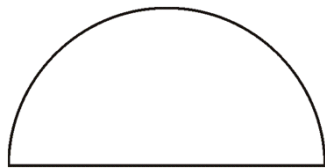
- Systems of mixed dynamics
- Scattering systems



# Bunimovich Mushroom Billiards: Classical Dynamics

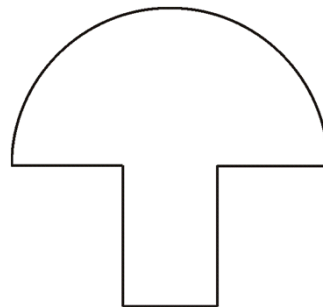
Variable width of stem:

Semi-circle



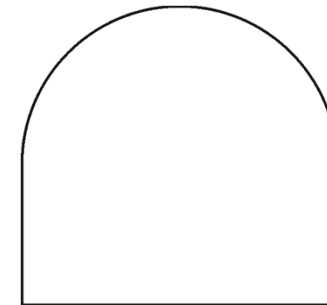
Regular

Mushroom



Mixed

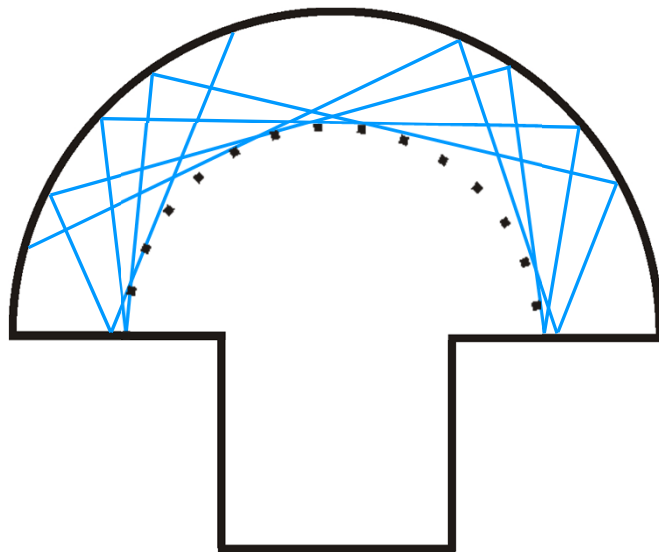
Stadium



Chaotic

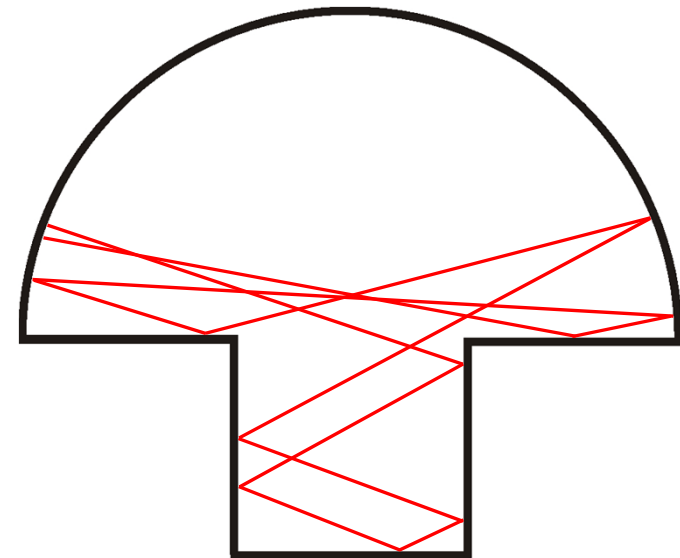
- Candidate for new standard system: generalized stadium

# Orbits in Mushroom Billiards



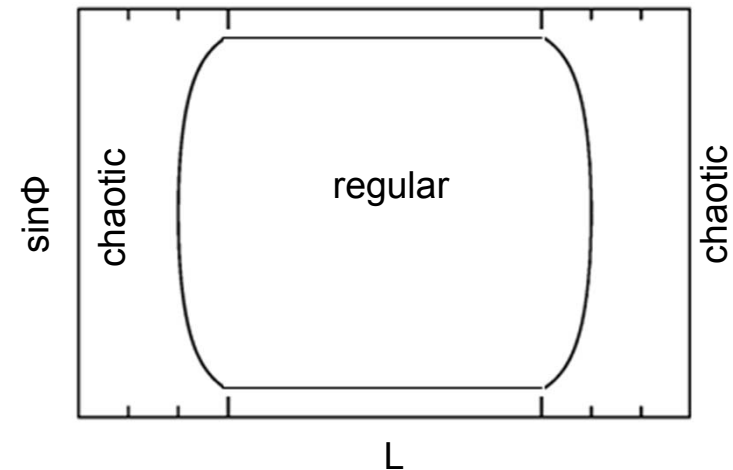
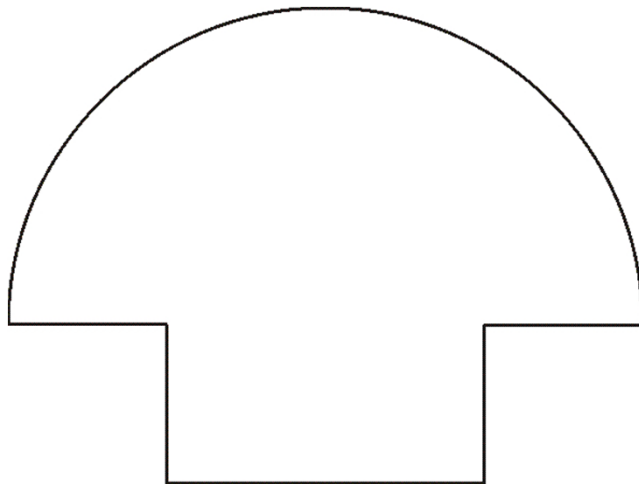
- Orbits with conservation of angular momentum in the hat  
→ **regular** dynamics
- Those orbits stay in the hat at all times

- Bunimovich: orbits which enter into the stem at some time  
→ **chaotic** dynamics



# Phase Space of Mushrooms

Poincaré section of a mushroom:



- Exact section (analytic, not simulated)
- Sharply divided areas in phase space
- No fractal properties

# Electromagnetic Cavities

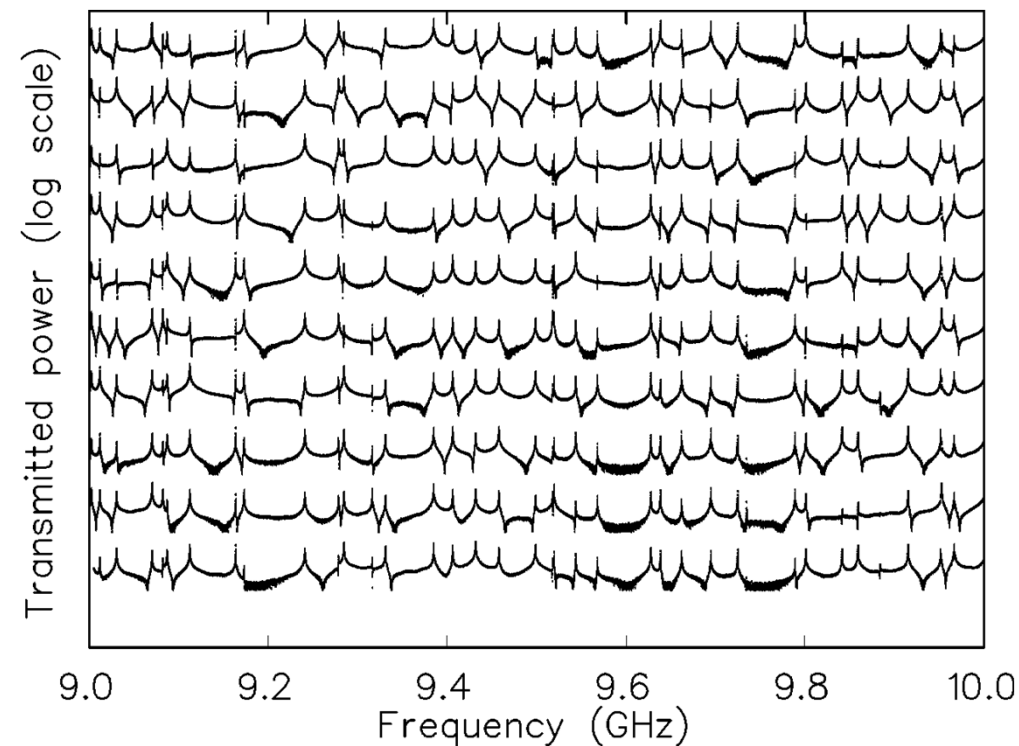


- Lead plated copper
- $Q \sim 10^5-10^6$
- 938 resonances found up to 22 GHz
- Spectrum complete so far (Weyl's formula)



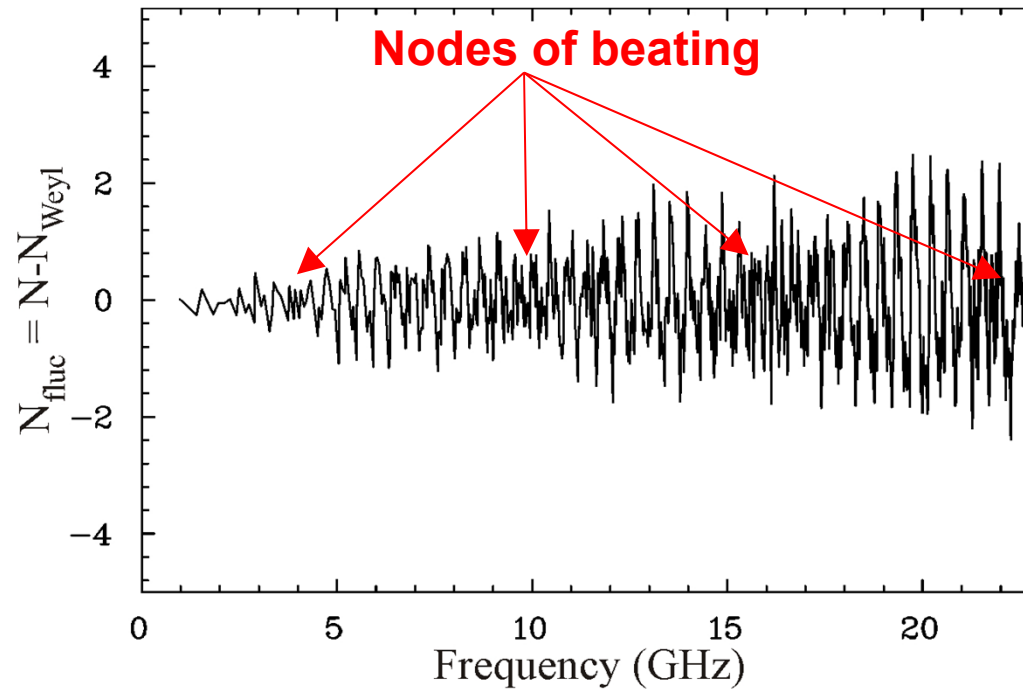
# Measured Spectra

Part of raw spectra from 10 antenna combinations



- Global fluctuations in the spacings: small and large gaps (“bunching”)

# Fluctuation Properties

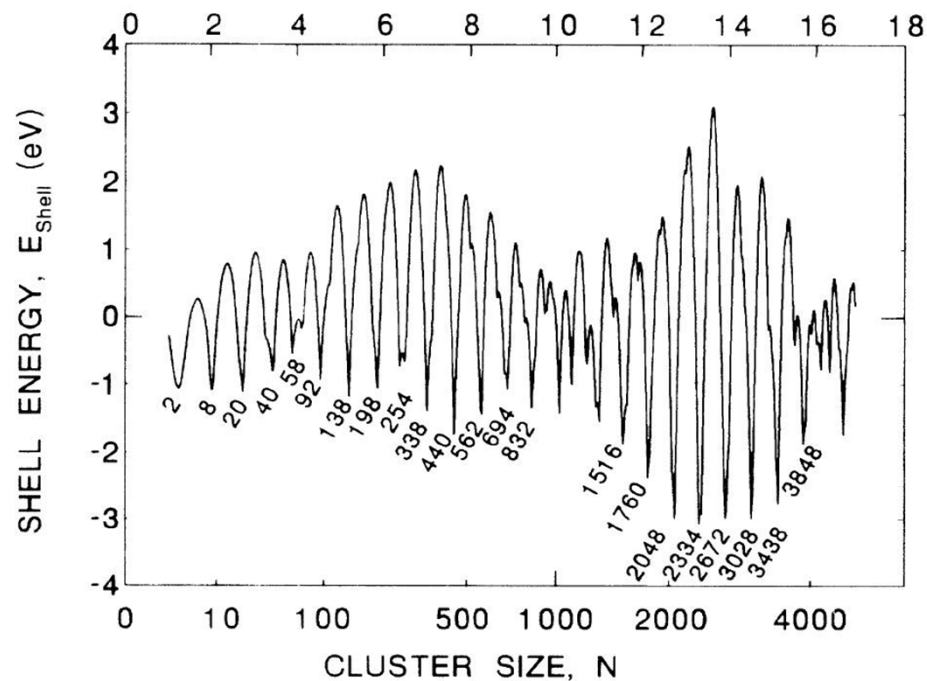


- Level number fluctuates: **oscillations** and **beating** → shell structures
- Do we see this behavior in other mesoscopic systems?

# Supershell Structures in Metal Clusters

- Consider the fluctuating part of the binding energy of valence electrons in a Na cluster

$$E(N) = \sum_{j=1}^N E_j = E_{av}(N) + E_{Shell}(N)$$



- Scale:  $10^{-7}$ - $10^{-6}$  m

Bjørnholm et al. (1990)

Nishioka, Hansen + Mottelson (1990)

# Periodic Orbit Theory (POT)

**Motivation:** description of quantum spectra in terms of classical closed orbits (Gutzwiller's trace formula)

spectrum

$$\{f_i\}$$



spectral density

$$\rho(f) = \sum_i \delta(f - f_i)$$



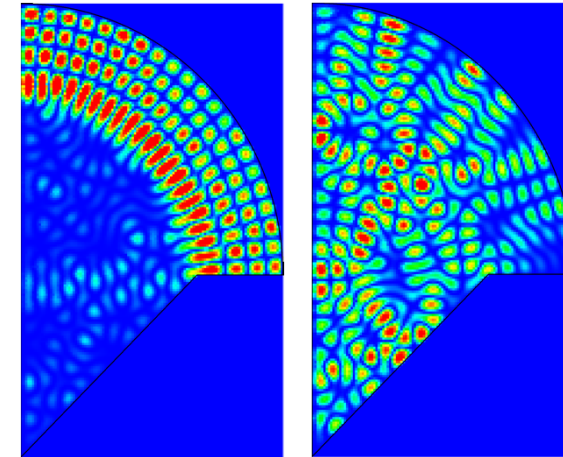
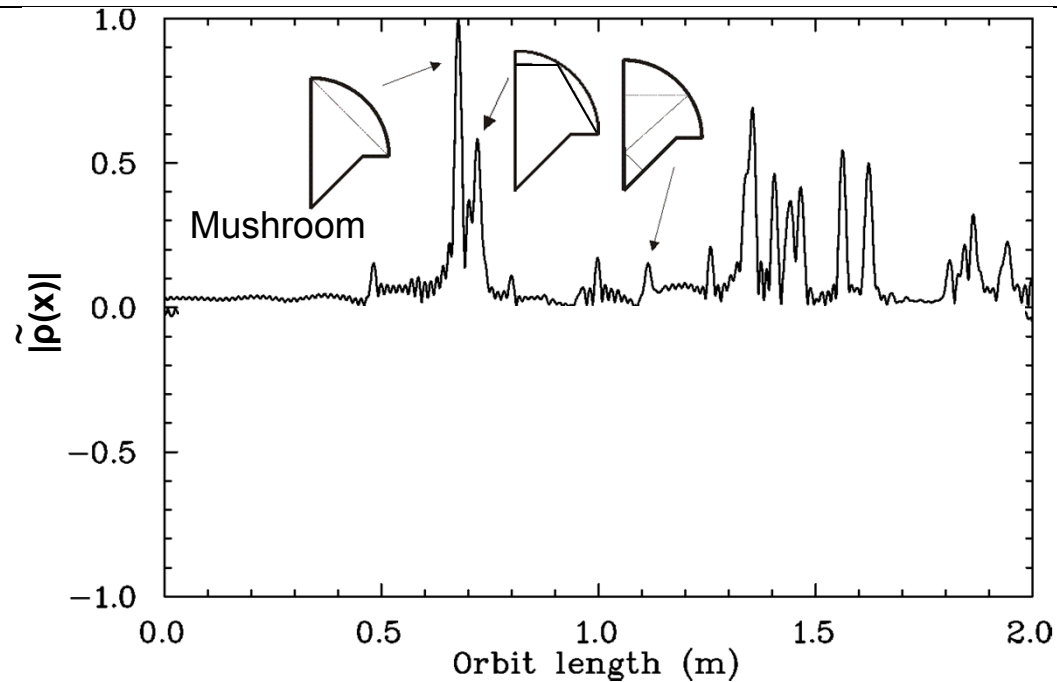
length spectrum

$$\tilde{\rho}(x) = \int_0^{f_{\max}} \rho(f) e^{i \frac{2\pi}{c} x f} df$$



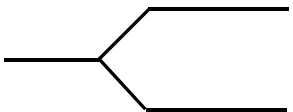
Peaks at the lengths  $x$  of PO's

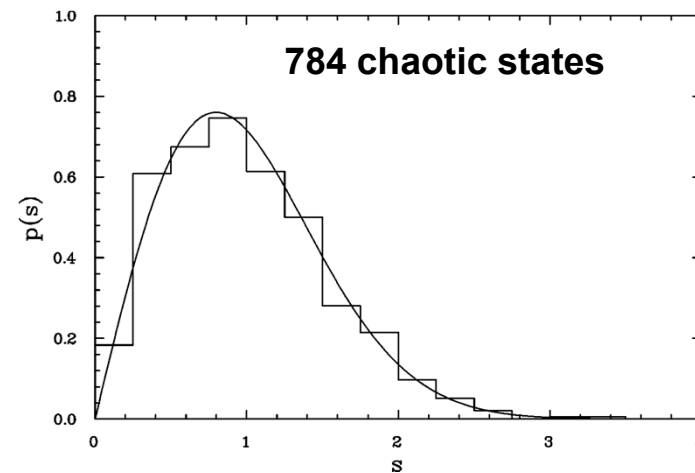
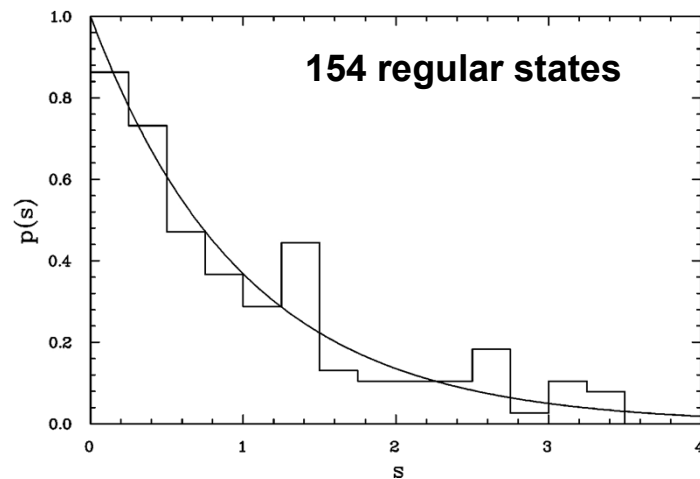
# Length Spectrum



- Part of the peaks are common to both the mushroom and the quarter circle  
→ regular orbits can be distinguished from the chaotic ones
- Isolated pair of peaks at  $\sim 0.7\text{m}$  causes the beating:  $\left| \text{□} + \text{○} \right|^2$
- Metal clusters:  $\left| \text{□} + \text{△} \right|^2$

# Classification of Eigenstates

- Spectrum  Subspectrum: Regular modes  $\rightarrow$  Poisson  
Subspectrum: Chaotic modes  $\rightarrow$  GOE



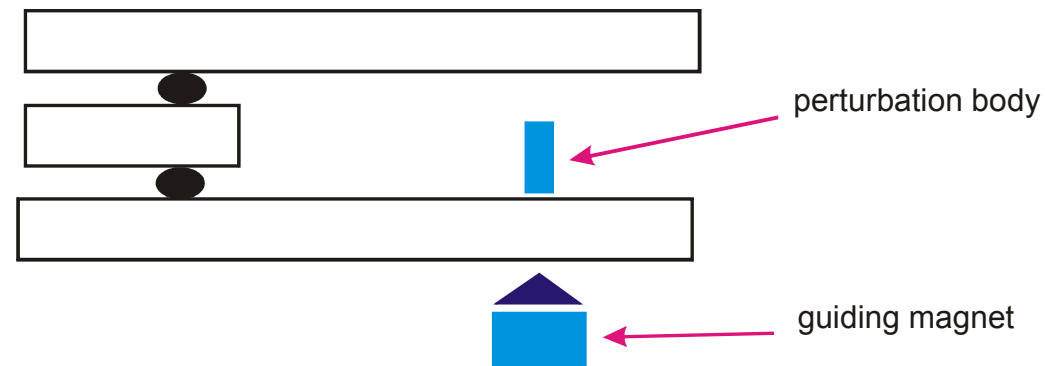
- Semiclassical limit reached for low quantum numbers
- Ratios 154/938 and 784/938 correspond to the respective parts of the phase space  
 $\rightarrow$  Percival's conjecture holds

# Cavities for Field Intensity Measurements



- XXL billiard  
→ get a quality factor  $Q \sim 10^4$   
even at room temperature
- Scaled copies of superconducting cavities

# Maier-Slater Theorem

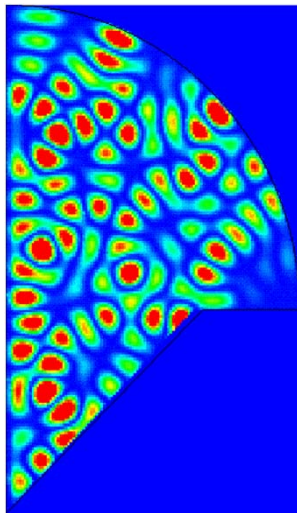


Dielectric perturbation body  
e.g. magnetic rubber

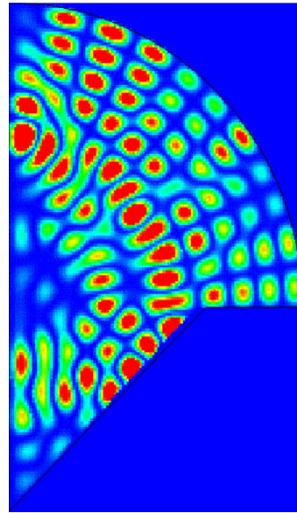
$$\delta f(x, y) = c_1 \cdot E^2(x, y) - \cancel{c_2} \cdot B^2(x, y)$$



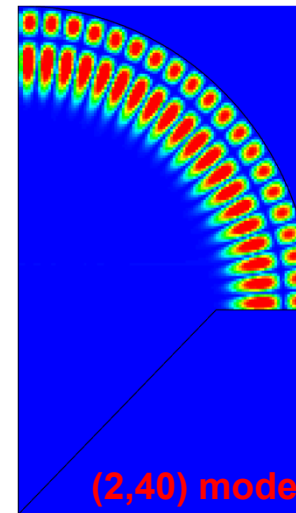
# Measured Intensity Distributions



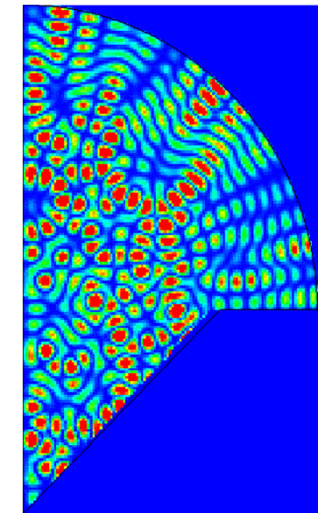
N~135



N~136



N~176

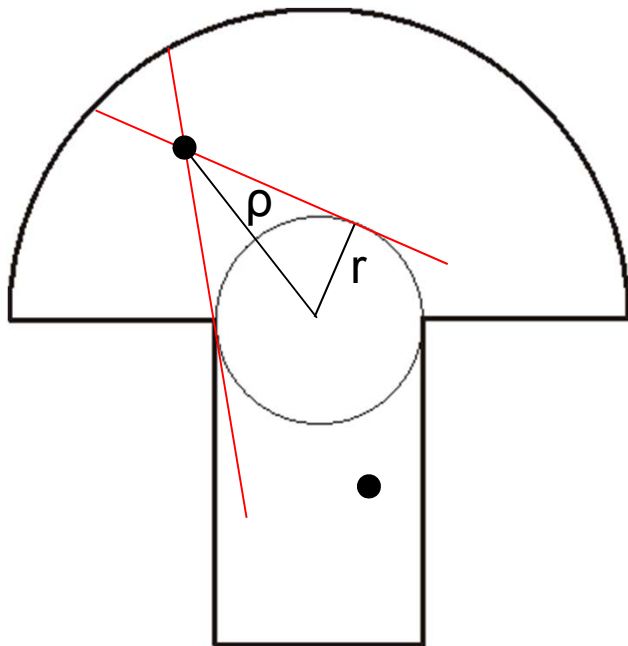


N~408

- Perturbation body method  $\rightarrow$  measure  $|E|^2 \sim |\psi|^2$
- Modes are **regular**, **chaotic** or (though rare) **mixed**
- Regular modes correspond to modes of a circular billiard

# Where in the Mushroom is how much Chaos?

- Regularity is restricted to the hat  $\rightarrow$  Less chaos in the hat!



- Orbits with angular momentum smaller than  $L_{\text{crit}}=r$  are chaotic

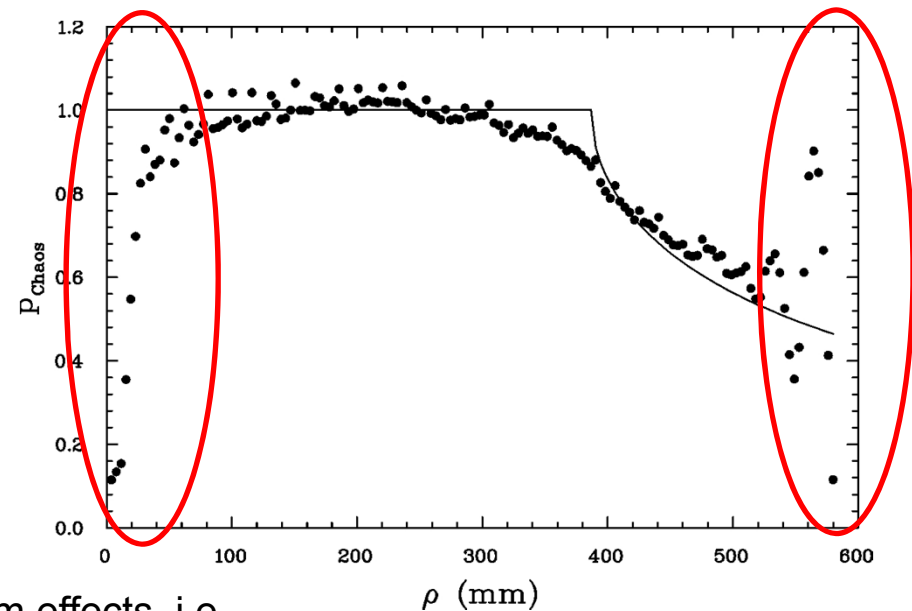
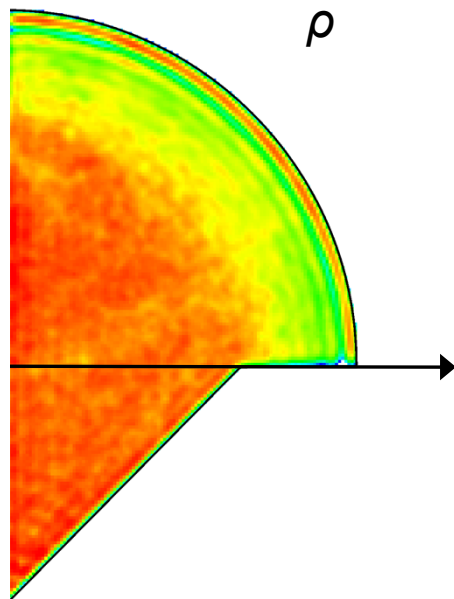
- $p_{\text{Chaos}}$  is constant in the stem, decreasing in the hat

$$p_{\text{Chaos}} = \begin{cases} 1 & \text{for } \rho < r \\ 2/\pi \cdot \arcsin(r/\rho) & \text{for } \rho > r \end{cases}$$

- Should be detectable in wave functions

# Averaged Intensity Distribution

Consider  $I(\vec{x}) = \sum_{\mu} |\Psi_{\mu}(\vec{x})|^2$  of 239 measured chaotic modes



- For  $\rho$  small,  $\rho$  large and at the “knee”  $\rightarrow$  quantum effects, i.e. deviations of order of the shortest  $\lambda$
- Behavior of averaged intensity distribution attributed to the separability of the classical phase space

# Friedel Oscillations

- The electronic wave function about an impurity atom in an alloy oscillates → Friedel oscillations

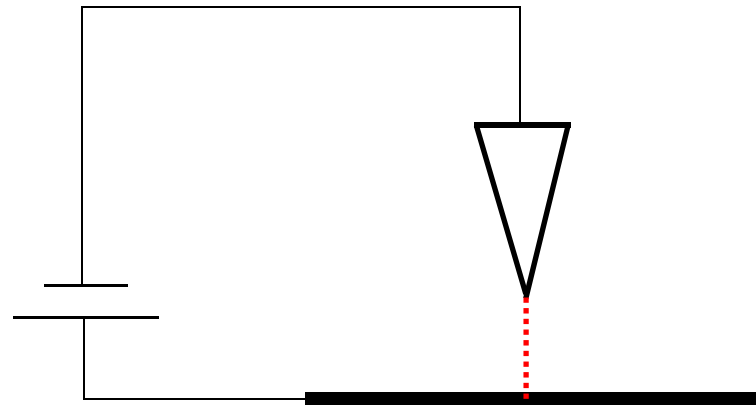
J. Friedel, Nuovo Cim. Suppl. **2**, 287 (1951)

- Oscillations also occur along spatial potential steps
- Electronic wave function on metallic surfaces are strongly influenced by defect atoms → oscillating behavior due to diffraction
- Oscillations might be studied through scanning tunneling microscopy

# Scanning Tunneling Microscopy

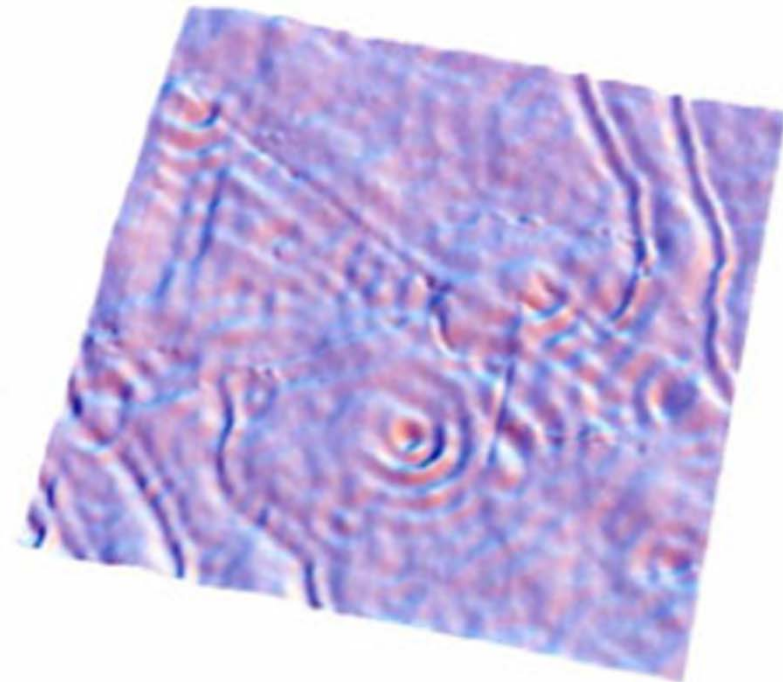
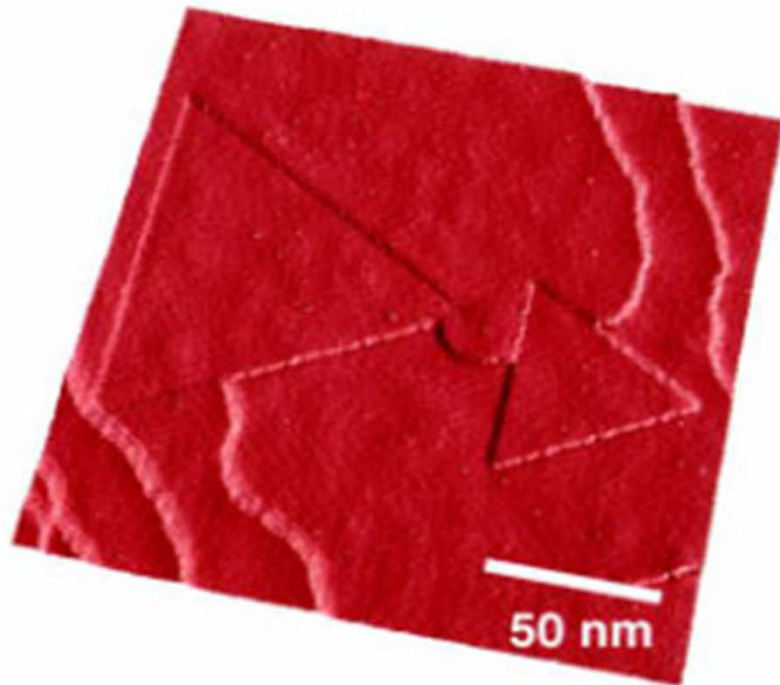
- Experimental approach: scanning tunneling microscopy

G.A. Fiete and E.J. Heller, Rev. Mod. Phys. **75**, 933 (2003)



- For cold surfaces, the tunnel current  $I(\vec{x}) \propto \sum_{\mu}^N |\Psi_{\mu}(\vec{x})|^2$
- Method to probe asymptotic properties of the wave functions

# Some Breathtaking Pictures



K. Kanisawa et al., Phys. Rev. Lett. **86**, 3384  
(2001)

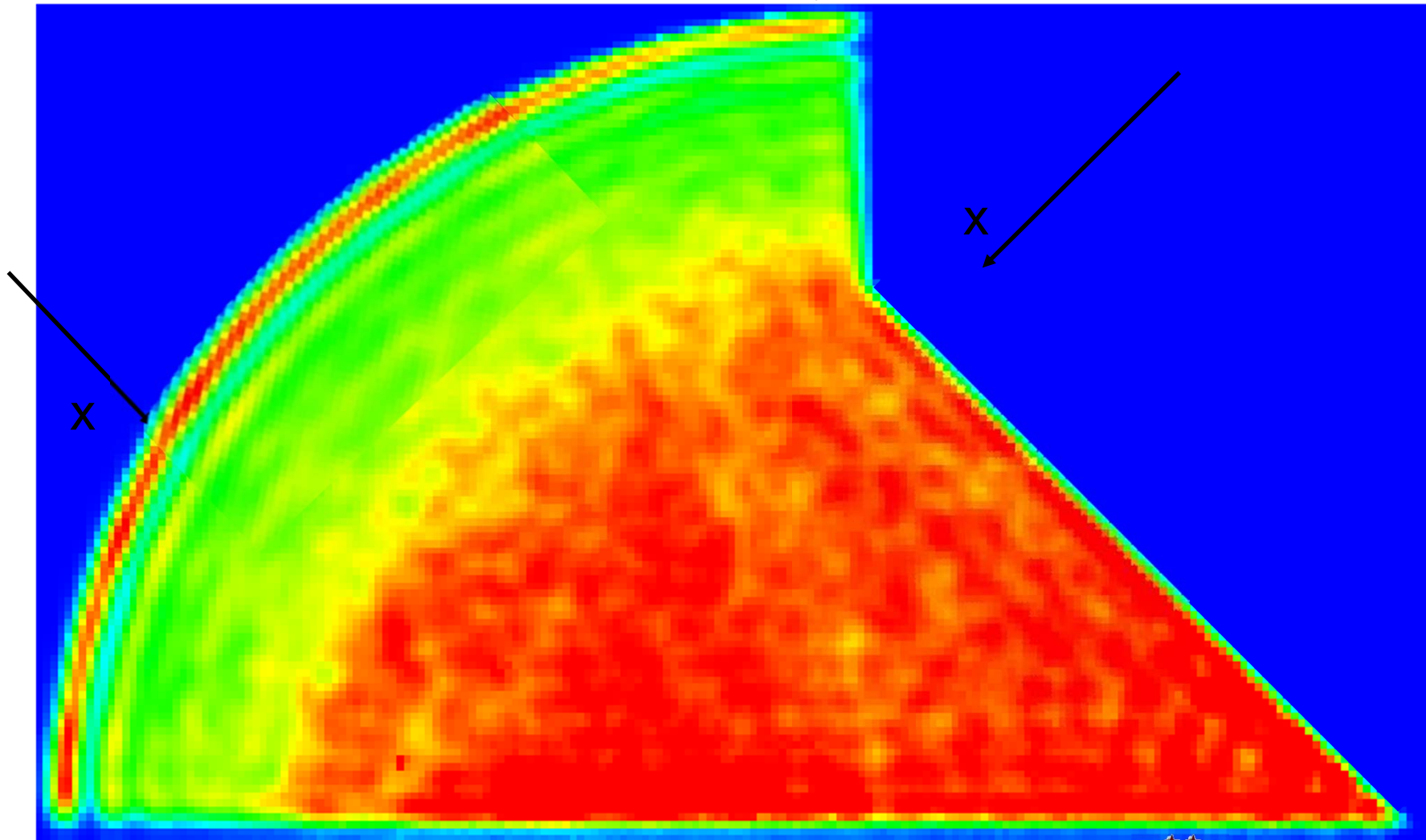
<http://www.almaden.ibm.com/vis/stm/corral.html>

# Friedel Oscillations in the Mushroom Billiard

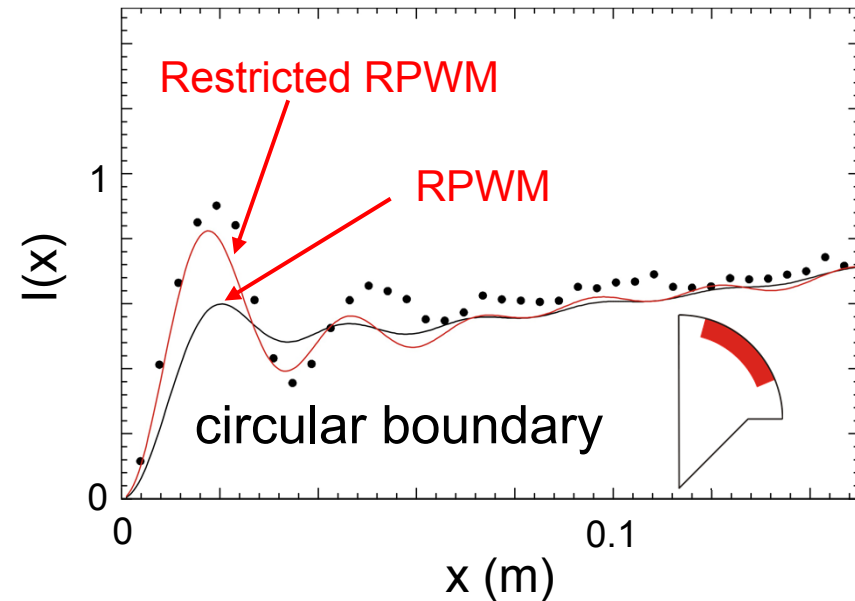
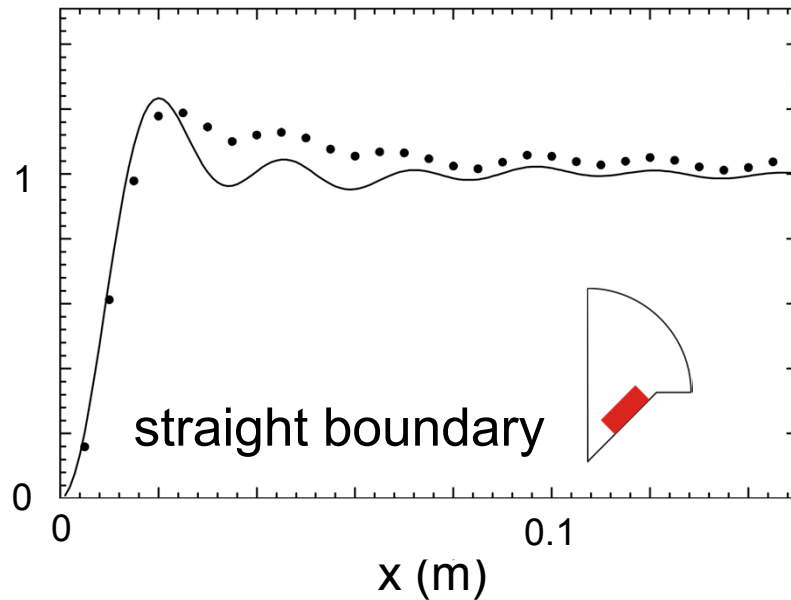


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- Superposition of 239 intensity distributions of chaotic modes



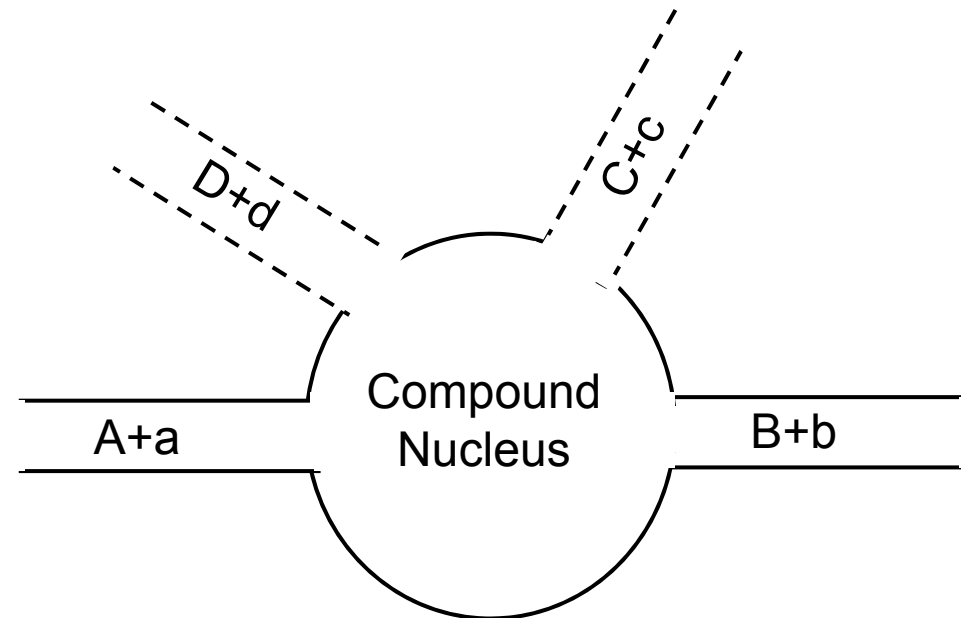
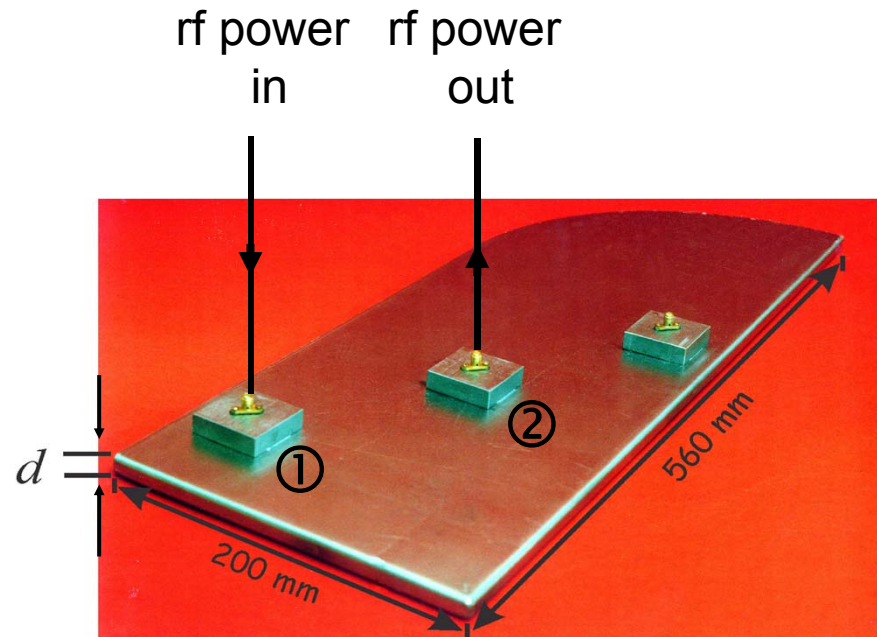
# Chaotic Modes in the Mushroom Billiard and RPWM



- Strong enhancement of oscillations along the circular boundary due to restricted accessible directions of interfering waves



# Microwave Resonator as a Model for the Compound Nucleus



- Microwave power is **emitted** into the resonator by antenna ① and the output signal is **received** by antenna ②  
→ **Open scattering system**
- The antennas act as **single scattering channels**
- Absorption into the walls is modelled by additive channels

# Scattering Matrix Description

- Scattering matrix for both scattering processes

$$\hat{S}(f) = I - 2\pi i \hat{W}^T \left( f I - \hat{H} + i\pi \hat{W} \hat{W}^T \right)^{-1} \hat{W}$$

## Compound-nucleus reactions

nuclear Hamiltonian

$\leftarrow \hat{H} \rightarrow$

coupling of quasi-bound states to channel states

$\leftarrow \hat{W} \rightarrow$

## Microwave billiard

resonator Hamiltonian

coupling of resonator states to antenna states and to the walls

- Experiment: complex S-matrix elements

- **RMT description:** replace  $\hat{H}$  by a **GOE** matrix for **T-inv** systems  
**GUE** **T-noninv**

# Decay of $S$ -matrix correlations

atomic nucleus

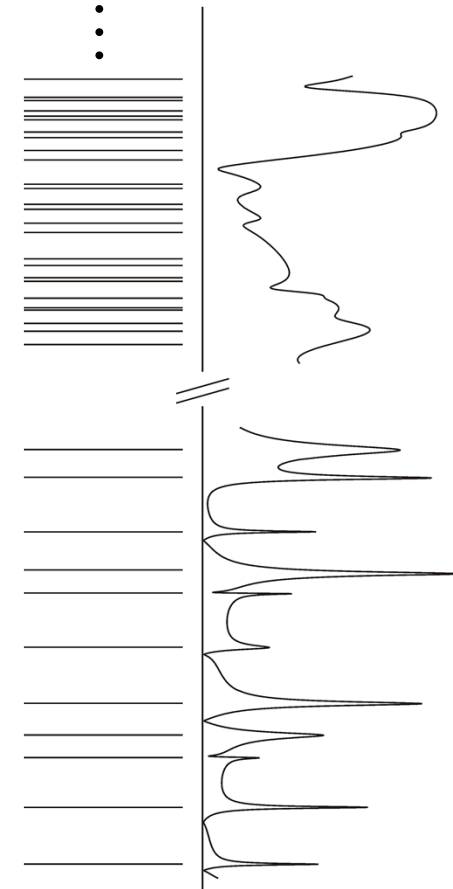


overlapping resonances  
for  $\Gamma/D > 1$   
**Ericson fluctuations**

isolated resonances  
for  $\Gamma/D \ll 1$

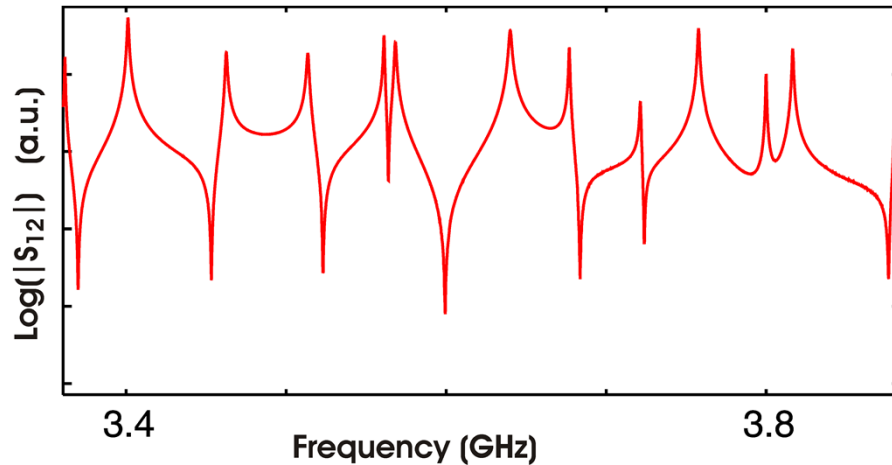
$$\rho \sim \exp(E^{1/2})$$

microwave cavity

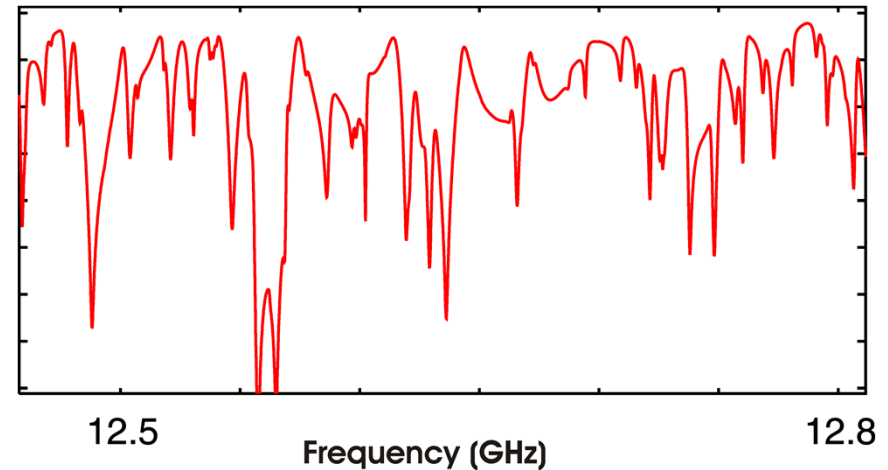


$$\rho \sim f$$

# Spectra and Autocorrelation Function



- Regime of isolated resonances
- $\Gamma/D$  small
- Resonances: eigenvalues, partial and total widths



- Overlapping resonances
- $\Gamma/D \sim 1$
- Fluctuations:  $\Gamma_{coh}$

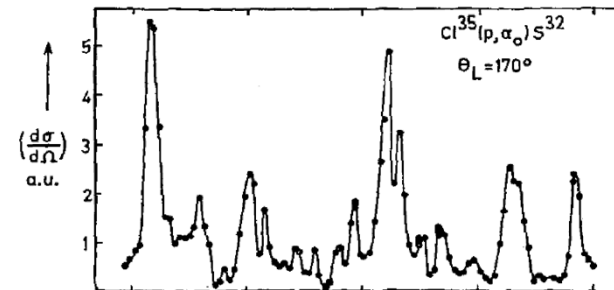
$$\text{Correlation function: } C(\varepsilon) = \langle S(f)S^*(f + \varepsilon) \rangle - \langle S(f) \rangle \langle S^*(f + \varepsilon) \rangle$$

# Ericson 's Prediction

- Ericson fluctuations (1960):

$$|C(\varepsilon)|^2 \propto \frac{\Gamma_{coh}^2}{\Gamma_{coh}^2 + \varepsilon^2}$$

- Correlation function is Lorentzian
- Measured 1964 for overlapping compound nuclear resonances
- Now observed in lots of different systems: molecules, quantum dots, laser cavities...
- Applicable for  $\Gamma/D \gg 1$  and for many open channels only



# Exact Random Matrix Theory Result

- For GOE systems and arbitrary  $\Gamma/D$  :  
Verbaarschot, Weidenmüller and Zirnbauer (VWZ) 1984

- VWZ-integral:

$$C = C(T_i, D; \varepsilon)$$

$$\overline{S_{ab}(f)S_{ab}^*(f + \epsilon)} = \frac{1}{8} \int_0^\infty d\lambda_1 \int_0^\infty d\lambda_2 \int_0^1 d\lambda \mu(\lambda, \lambda_1, \lambda_2)$$

**Transmission coefficients**

$$\times \exp(-i\pi\epsilon(\lambda_1 + \lambda_2 + 2\lambda)/D)$$

$$\times J_{ab}(\lambda, \lambda_1, \lambda_2)$$

$$\times \prod_e \frac{(1 - T_e\lambda)}{((1 + T_e\lambda_1)(1 + T_e\lambda_2))^{1/2}}$$

$$\mu(\lambda, \lambda_1, \lambda_2) = \frac{\lambda(1 - \lambda)|\lambda_1 - \lambda_2|}{(\lambda + \lambda_1)^2(\lambda + \lambda_2)^2(\lambda_1\lambda_2(1 + \lambda_1)(1 + \lambda_2))^{1/2}}$$

**Average level distance**

$$J_{ab}(\lambda, \lambda_1, \lambda_2) = \delta_{ab} T_a^{1/2} (1 - T_a)$$

$$\times \left( \frac{\lambda_1}{1 + T_a\lambda_1} + \frac{\lambda_2}{1 + T_a\lambda_2} + \frac{2\lambda}{1 - T_a\lambda} \right)$$

$$+ (1 + \delta_{ab}) T_a T_b$$

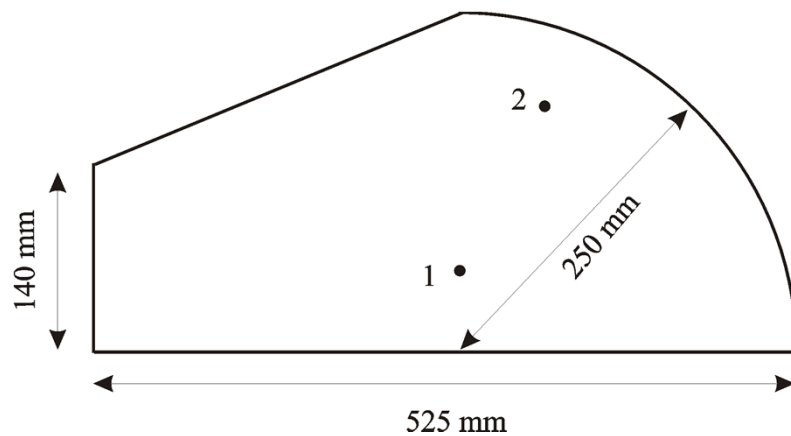
$$+ \left( \frac{\lambda_1(1 + \lambda_1)}{(1 + T_a\lambda_1)(1 + T_b\lambda_1)} + \frac{\lambda_2(1 + \lambda_2)}{(1 + T_a\lambda_2)(1 + T_b\lambda_2)} \right)$$

$$+ \frac{2\lambda(1 - \lambda)}{(1 - T_a\lambda)(1 - T_b\lambda)}$$

- Rigorous test of VWZ: isolate
- Our goal: test VWZ in the intermediate regime, i.e.  $\Gamma/D \approx 1$

# Spectra of $S$ -Matrix Elements in a fully Chaotic Tilted Stadium Billiard

Example: 8-9 GHz

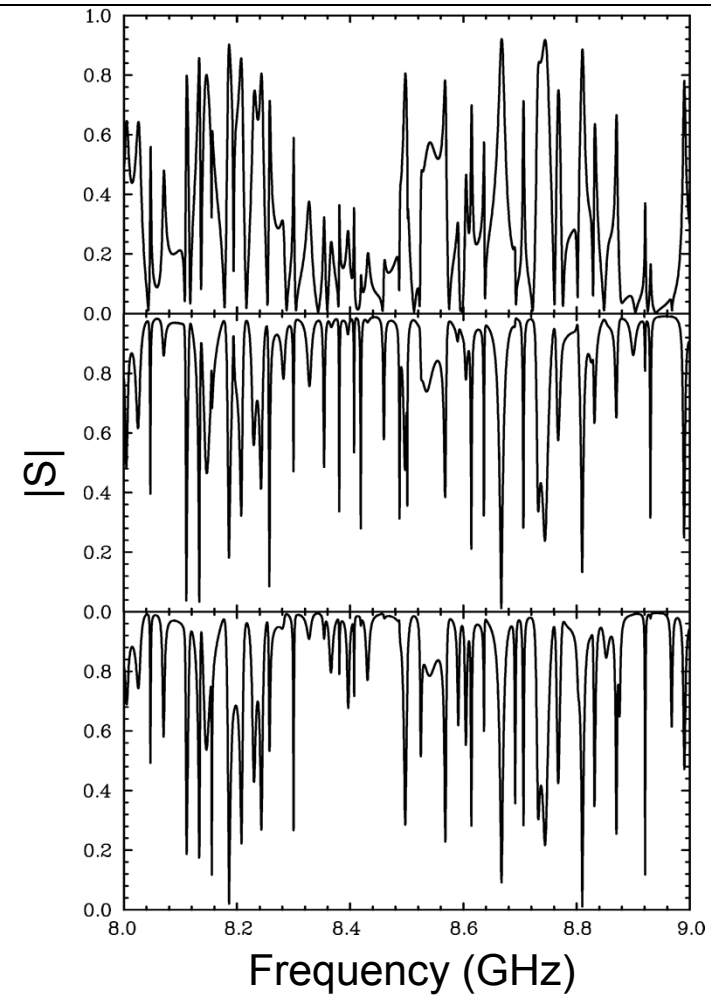


- T-invariant tilted stadium billiard

$S_{12}$  →

$S_{11}$  →

$S_{22}$  →



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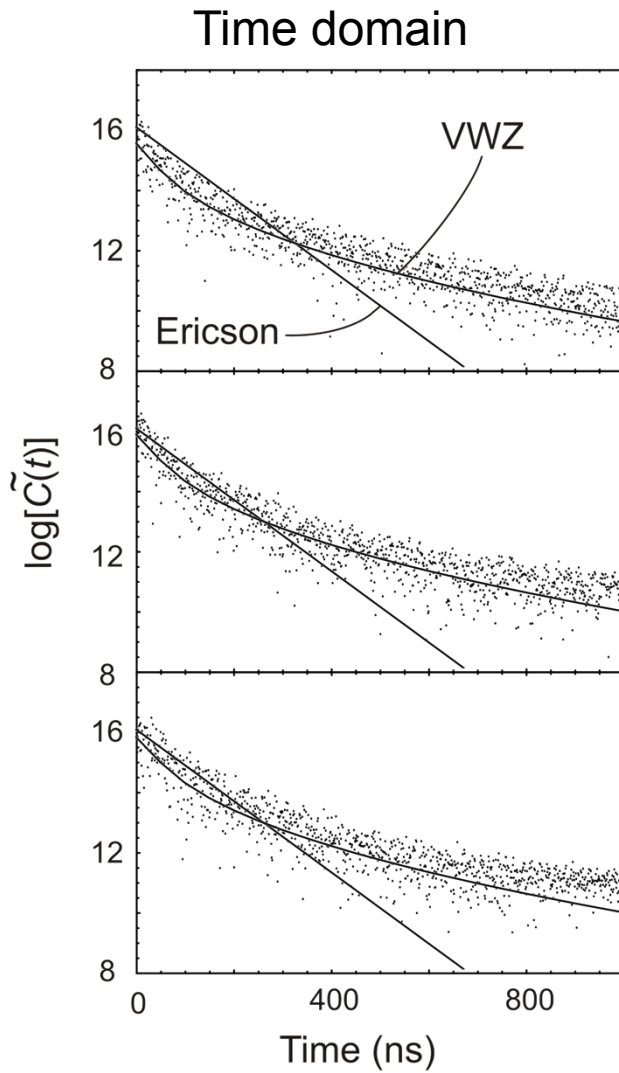
# Road to Analysis

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- Problem: adjacent points in  $C(\varepsilon)$  are correlated
- Solution: FT of  $C(\varepsilon) \rightarrow$  uncorrelated Fourier coefficients  $C(t)$   
Ericson (1965)
- Development: Non Gaussian fit and test procedure



# Comparison: Experiment vs. VWZ

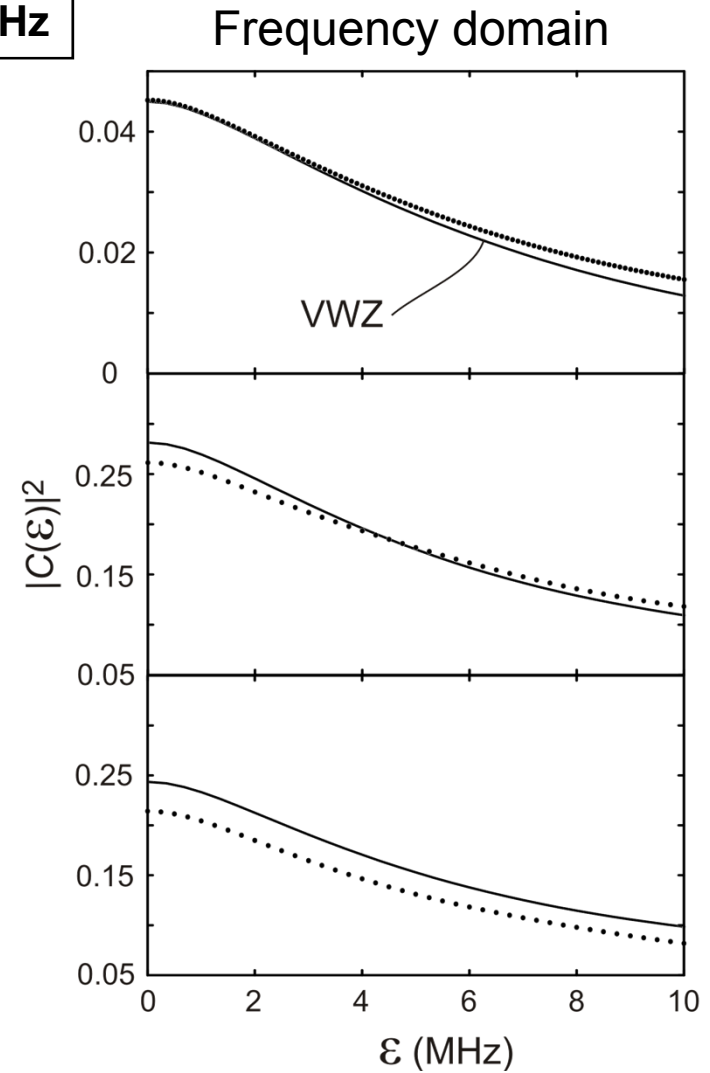


Example 8-9 GHz

←  $S_{12}$

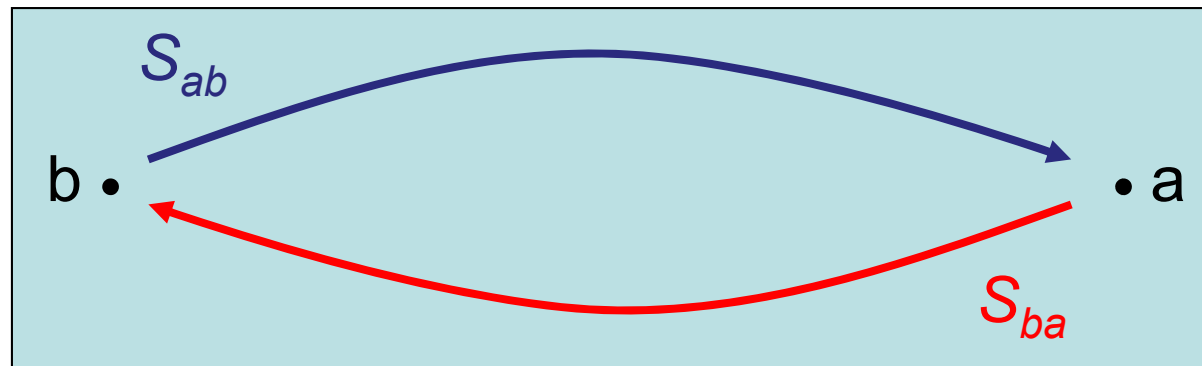
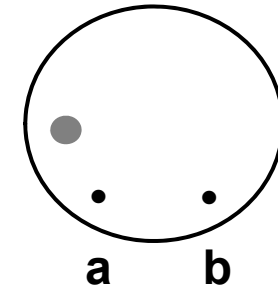
←  $S_{11}$

←  $S_{22}$



# Induced Time-Reversal Symmetry Breaking (TRSB) in Billiards

- T-symmetry breaking caused by a magnetized ferrite
- Coupling of microwaves to the ferrite depends on the direction  $a \leftrightarrow b$



- Principle of detailed balance:  $|S_{ab}|^2 = |S_{ba}|^2$
- Principle of reciprocity:  $S_{ab} = S_{ba}$

# Search for TRSB in Nuclei: Ericson Regime



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1 AUGUST 1983

## Improved Experimental Test of Detailed Balance and Time Reversibility in the Reactions $^{27}\text{Al}+p \rightleftharpoons ^{24}\text{Mg} + \alpha$

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and

H. Genz, A. Richter, and G. Schrieder

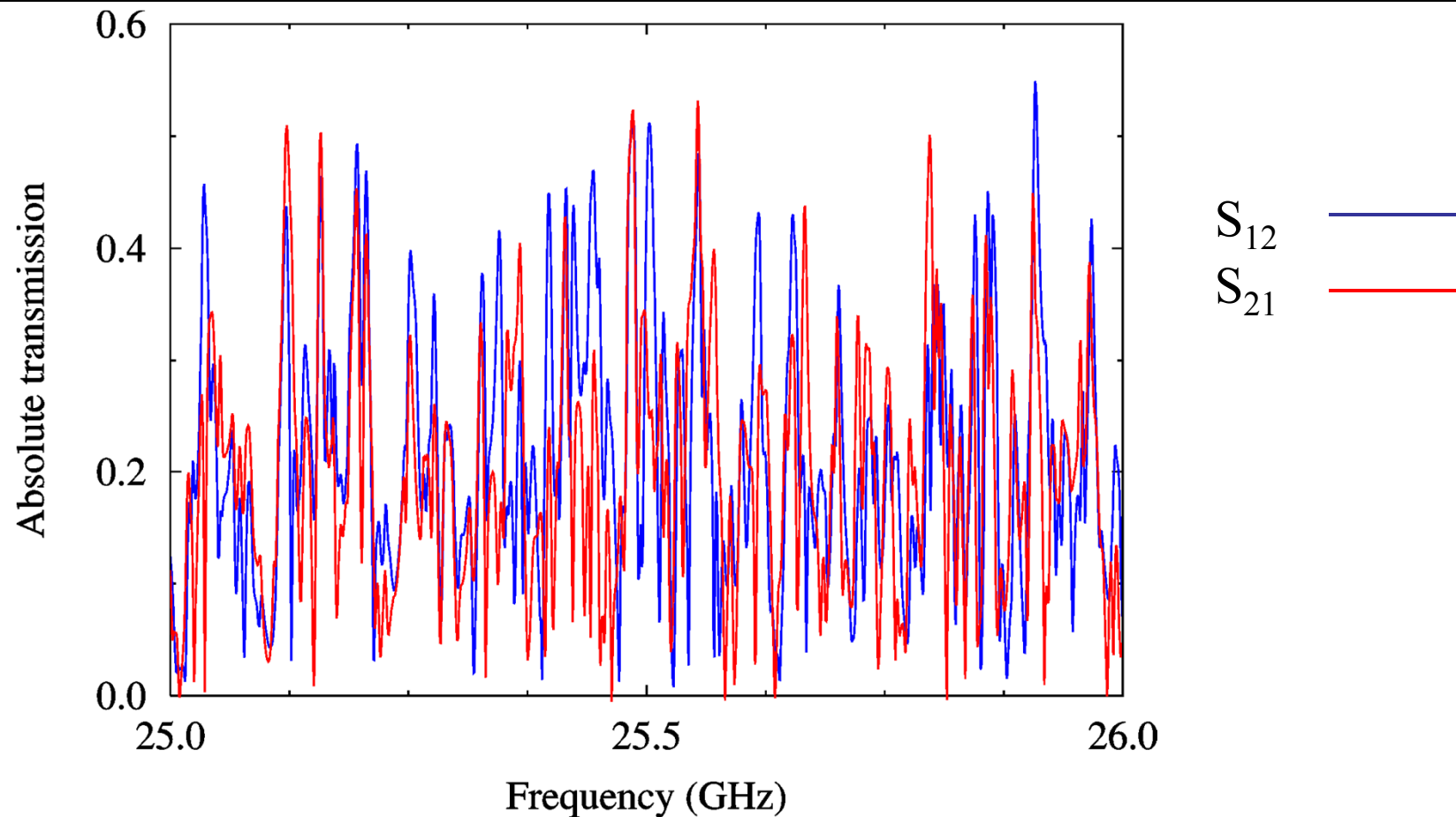
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(Received 25 April 1983)

A new test of the principle of detailed balance in the nuclear reactions  $^{27}\text{Al}(p, \alpha_0) ^{24}\text{Mg}$  and  $^{24}\text{Mg}(\alpha, p_0) ^{27}\text{Al}$  at bombarding energies  $7.3 \text{ MeV} \leq E_p \leq 7.7 \text{ MeV}$  and  $10.1 \text{ MeV} \leq E_\alpha \leq 10.5 \text{ MeV}$ , respectively, is reported. Measured relative differential cross sections agree within the experimental uncertainty  $\Delta = \pm 0.51\%$  and hence are consistent with time-reversal invariance. From this result an upper limit  $\xi \leq 5 \times 10^{-4}$  (80% confidence) is derived for a possible time-reversal-noninvariant amplitude in the reaction.



# Violation of Reciprocity



- Clear violation of reciprocity in the regime of  $\Gamma/D \approx 1$

# Analysis of Fluctuations with Crosscorrelation Function

Crosscorrelation function:

$$C(S_{12}, S_{21}^*, \varepsilon) = \langle S_{12}(f) S_{21}^*(f + \varepsilon) \rangle - \langle S_{12}(f) \rangle \langle S_{21}^*(f) \rangle$$

- Determination of T-breaking strength from the data
- Special interest in first coefficient ( $\varepsilon = 0$ )

# Exact RMT Result for Partial T-Breaking

- RMT analysis based on Pluhař, Weidenmüller, Zuk, Lewenkopf and Wegner, 1995

$$C_{ab}(\epsilon) = \frac{T_a T_b}{16} \int_0^\infty d\mu_1 \int_0^\infty d\mu_2 \int_0^1 d\mu \frac{C(T_1, T_2, \tau_{abs}; D, \epsilon, \xi)}{\mathcal{U}}$$

$$\times \frac{1}{(\mu + \mu_1)^2} \frac{1}{(\mu + \mu_2)^2} \exp\left(-\frac{i\pi\epsilon}{D}(\mu_1 + \mu_2 + 2\mu)\right)$$

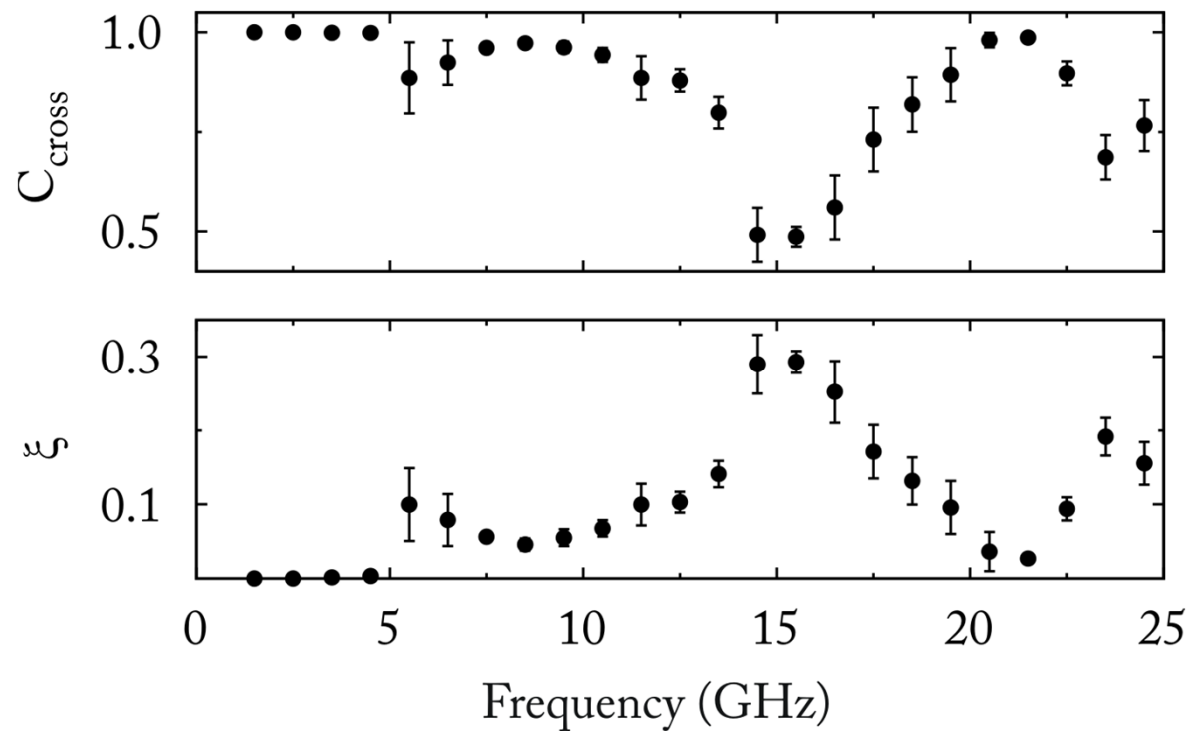
$$\times J_{ab} \cdot \prod_c \frac{1 - T_c \mu}{\sqrt{(1 + T_c \mu_1)(1 + T_c \mu_2)}} \exp(-2\mathfrak{t}\mathcal{H}),$$

$$K_{ab} = \epsilon_- \left[ 2\mathcal{F} \left\{ (\tilde{A}_a \tilde{C}_b + \tilde{A}_b \tilde{C}_a) \mathcal{G} \lambda_2 + (\tilde{B}_a \tilde{C}_b + \tilde{B}_b \tilde{C}_a) \mathcal{H} \lambda_1 \right\} \right. \\ \left. + 3C_3 \mathcal{F} - C_2 (\lambda_2^2 - \lambda_1^2) + C_2 \mathfrak{t}\mathcal{R} (4\lambda_2^2 - 2\mathcal{F}) \right. \\ \left. + (\epsilon_+ - \frac{\epsilon_-}{\mathfrak{t}\mathcal{F}}) \left[ 3C_3 (\lambda_2^2 - \lambda_1^2) + \mathfrak{t}\mathcal{R} C_3 (4\lambda_2^2 - 2\mathcal{F}) \right. \right. \\ \left. \left. + 2\mathcal{F} \left\{ (\tilde{A}_a \tilde{C}_b + \tilde{A}_b \tilde{C}_a) \mathcal{G} \lambda_2 - (\tilde{B}_a \tilde{C}_b + \tilde{B}_b \tilde{C}_a) \mathcal{H} \lambda_1 \right\} \right. \right. \\ \left. \left. + (2\mathfrak{t}\mathcal{R} - 1) C_2 \mathcal{F} \right]. \right.$$

**T-symmetry breaking parameter**

$$J_{ab} = \left\{ \left[ \left( \frac{1}{2} \frac{\mu_1(1 + \mu_1)}{(1 + T_a \mu_1)(1 + T_b \mu_1)} + \frac{1}{2} \frac{\mu_2(1 + \mu_2)}{(1 + T_a \mu_2)(1 + T_b \mu_2)} \right. \right. \right. \\ \left. \left. + \frac{\mu(1 - \mu)}{(1 - T_a \mu)(1 - T_b \mu)} \right) (1 + \delta_{ab}) \right. \\ \left. \left. + 2\delta_{ab} \overline{S_{aa}^2} \left( \frac{\mu_1}{2(1 + T_a \mu_1)} + \frac{\mu_2}{2(1 + T_a \mu_2)} + \frac{\mu}{1 - T_a \mu} \right)^2 \right] \right. \\ \left. \times [\mathcal{F} \epsilon_+ + (\lambda_2^2 - \lambda_1^2) \epsilon_- + 4\mathfrak{t}\mathcal{R} (\lambda_2^2 + \mathcal{F} (\epsilon_+ - 1))] \right. \\ \left. \pm (1 - \delta_{ab}) K_{ab} \right\} + \{\lambda_1 \rightleftharpoons \lambda_2\}$$

# Determination of T-Breaking Strength



- B. Dietz *et al.*, PRL **103**, 064101 (2009).

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# Summary

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- **Quantum chaos** manifests itself in the universal (generic) behavior of **spectral properties** and **wave functions and scattering matrix elements**.
- **Analogue experiments with microwave resonators** provide interesting models for mesoscopic systems.



„Wo das Chaos auf die Ordnung trifft,  
gewinnt meist das Chaos, weil es besser  
organisiert ist.“

”Where chaos meets order, wins mostly  
chaos, since it is better organized.“

Friedrich Nietzsche (1844-1900)

